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GYOLDER FAZOLARIDA KOMPAKT OPERATORLAR

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Annotatsiya

Yadrosining maxsuslik chizig'i integrallash chizig'ini o'ziga akslantiruvchi funksiya (siljish) orqali berilganda singulyar integral tenglamalar nazariyasi qaralayotgan funksiyalar fazosi, integrallash chizig'i turlari ularning akslantirishlari ko'rinishlariga bog'liq ravishda olib borilgan tadqiqotlar ko'lami ancha keng bo'lib, ular haqidagi ma'lumotlar [1] da batafsil keltirilgan.
Ma'lum Banax fazosida aniqlangan, obrazi yopiq bo'lib, yadrosi va koyadrosi o'lchovlari chekli bo'lgan chiziqli chegaralangan operatorlar sinfi Nyoter operatorlari deb ataladi. Qaralayotgan operatorning yadrosi va koyadrosi ayirmasi shu operatorning indeksi deyiladi.
Siljishli singulyar integral operatorlarni Nyoter operator bo'lishlikka tekshirishda avvalgi tadqiqotlardan farqli o'laroq, integrallash chzig'ining o'zini o'ziga akslantirish funksiyas(siljish)i diffeomorfizm emas balki endomorfizm (ko'p varaqli) bo'lgan hol uchun muhim hisoblangan, siljish funksiyasi yordamida tuzilgan ba'zi operator-kommutatorlarni qaralayotgan Banax fazolarida kompaktligi muhim hisoblanadi.
Ushbu maqolada ko'rsatilgan Banax fazosida ikkita komutatorning ma'lum shartlarda kompakt operatorlar ekanligi isbotlangan.

Kalit so'zlar: *singulyar, integral, siljish, diffeomorfizm, endomorfizm, Gyolder fazosi, kommutator, siljish operatori, kompaktlik, norma.*

Faraz qilaylik Γ-kompleks tekislikda oddiy yopiq, silliq, yo'naltirilgan chiziq bo'lib, uni diametri d bo'lsin. [4] ga ko'ra, Φ orqali (0,d] da aniqlangan, aynan nolga teng bo'lmagan, nomanfiy va

a) ω(δ)-uzluksizlik moduli,

b) $\sup\limits_{\delta\in(0,d]} \frac{\delta}{\omega(\delta)}\int_0^d \frac{\omega(t)}{t(t+\delta)}dt < +\infty$

shartlarni qanoatlantiruvchi ω(δ) funksiyalar sinfini belgilaymiz.

Γ da uzluksiz funksiyalarni xarakteristikasi sifatida

$$\omega(\varphi,\delta) = \sup_{|t-\tau|\le\delta} |\varphi(t)-\varphi(\tau)|, \delta \in (0,d] \qquad (1)$$

uzluksizlik modulini tanlaymiz.

[4] ga ko'ra umumlashgan Gyolder fazolari deb ataluvchi $H_\omega(\Gamma), \omega \in \Phi$ *da aS – SaI* va *WS –SW* kommutatorlarni qaraymiz. Bu yerda $a(t) \in H_\omega(\Gamma)$, *S* –singulyar integral operator bo'lib, u

$$(S\varphi)(t)=\frac{1}{\pi i}\int_\Gamma \frac{\varphi(t)}{\tau-t}d\tau, t \in \Gamma \qquad (2)$$

tenglik bilan aniqlanadi. W esa bir varaqli bo'lmagan siljish operatori bo'lib, u (Wφ)(t)=φ(α(t)) tenglik bilan aniqlanib, α (t) Γni o'ziga akslantiruvchi N (2≤N<∞) varaqli akslantirish bo'lib, α′(t)≠0, $t \in \Gamma$, $\alpha'(t) \in H_\omega(\Gamma), \omega \in \Phi$, ([3] ga qarang). Ta'rif bo'yicha ([4]ga qarang) Γ da aniqlangan $\varphi(t)$ funksiya

$$\operatorname*{Sup}_{0<\delta\leq d} \frac{\omega(\varphi,\delta)}{\omega(\delta)} = C_\varphi \leq +\infty \qquad (3)$$

shartni qanoatlantirsa, H_ω sinfga qarashli deyiladi. Ma'lumki, agar $\varphi(t) \in H_\omega$ bo'l,sa

$$C_\varphi = \operatorname*{Sup}_{\substack{t,\tau\in\Gamma\\ t\neq\tau}} \frac{\varphi(t)-\varphi(\tau)}{\omega(|t-\tau|)} = C_\varphi^* \qquad (4)$$

tenglik o'rinli bo'ladi.

Xususiy holda, agar $\omega(\delta)=\delta^\mu$ $(0<\mu<1)$ bo'lsa, H_ω bizga ma'lum bo'lgan H_μ sinfga aylanadi.

H_ω da normani

$$\|\varphi\|_{H_\omega} = \|\varphi\|_{C(\Gamma)} + \operatorname*{Sup}_{0<\delta\leq d} \frac{\omega(\varphi,\delta)}{\omega(\delta)} \qquad (5)$$

ko'rinishda belgilasak, H_ω banax fazosidan iborat bo'ladi.

S singulyar va integral operatorning $H_\omega(\Gamma)$ da chegaralanganligi [6] da isbotlangan. Bir varaqli bo'lmagan W siljish operatorining $H_\omega(\Gamma)$ da chegaralanganligi uni quyidagi baholanishidan kelib chiqadi:

$$\|W\varphi\|_{H_\omega(\Gamma)} = \max_{t\in\Gamma} |\varphi[\alpha(t)]| + \operatorname*{Sup}_{\substack{t,\tau\in\Gamma\\ t\neq\tau}} \frac{|\varphi[\alpha(t)]-\varphi[\alpha(\tau)]|}{\omega(|t-\tau|)} \leq$$

$$\leq \max_{t\in\Gamma} |\varphi(t)| + + \operatorname*{Sup}_{\substack{t,\tau\in\Gamma\\ t\neq\tau}} \frac{|\varphi(t)-\varphi(\tau)|}{\omega(|t-\tau|)} \cdot \operatorname*{Sup}_{\substack{t,\tau\in\Gamma\\ t\neq\tau}} \omega \frac{\omega(|\alpha(t)-\alpha(\tau)|)}{\omega(|t-\tau|)} \leq$$

$$\leq \max\left\{1;\ \operatorname*{Sup}_{\substack{t,\tau\in\Gamma\\ t\neq\tau}} \omega \frac{\omega(|\alpha(t)-\alpha(\tau)|)}{\omega(|t-\tau|)}\right\} \cdot \|\varphi\|_{H_\omega(\Gamma)}$$

Bu tenglikni hisobga olgan holda,

$\omega(|\alpha(t)-\alpha(\tau)|) \leq \omega(k\,|t-\tau|) \leq (k+1)\omega(|t-\tau|),\ k-$ o'zgarmas son. Tengsizlikka ko'ra W siljish operatori normasi uchun

$\|W\|_{H_\omega(\Gamma)} \leq \max\left\{1;\ \operatorname*{Sup}_{\substack{t,\tau\in\Gamma\\ t\neq\tau}} \omega \frac{\omega(|\alpha(t)-\alpha(\tau)|)}{\omega(|t-\tau|)}\right\}$ tengsizlikni hosil qilamiz.

V_n orqali Γ ni o'zini o'ziga bir varaqli bo'lmagan α(t), t∈Γ, akslantirishga o'ngdan teskarilanuvchi bo'lgan $\bar{\alpha}(t)$= <n,t> akslantirish yordamida hosil bo'lgan siljish operatorini belgilaymiz: $(V_n\,\varphi)(t) = \varphi[\bar{\alpha}(t)] = \varphi(<n,t>)$.

Shuni ta'kidlash lozimki, ([3] ga qarang), bir varaqli bo'lmagan siljish operatori W faqat chapdan teskarilanuvchidir. Aniqrog'i α Γni qoplovchi akslantirish bo'lgani uchun $\frac{1}{N}\sum_{n=1}^{N} V_n$ operator $H_\omega(\Gamma)$ fazoni o'ziga o'tkazadi va $(\frac{1}{N}\sum_{n=1}^{N} V_n)W = I$ (I-birlik operator) tenglik bajarilib, W o'lchovi cheksiz bo'lgan koyadroga ega bo'ladi.

Qaralayotgan $H_\omega(\Gamma)$ fazoda $aS - SaI$ kommutatorning kompaktligi [5] maqolada isbotlangan. Bir varaqli bo'lmagan $\alpha(t)$, t ∈ Γ akslantirish yordamida qaralgan siljish operatori W bilan tuzilgan WS-SW kommutatorning kompaktligini isbotlash uchun quyidagi yordamchi tasdiqni qaraymiz. Faraz qilaylik, z=σ(t) Γni birlik aylanaga o'zaro bir qiymatli akslantiruvchi funksiya bo'lsin. S_Γ va S_T lar mos ravishda $\mathrm{H}_\omega(\Gamma)$ va $\mathrm{H}_\omega(T)$ fazolardagi singulyar integrallash operatorlari bo'lsin. B bilan $\mathrm{H}_\omega(\Gamma)$ fazodan $\mathrm{H}_\omega(T)$ fazoga (Bφ)(z)=φ($\frac{\sigma^{-1}(z)}{}$), $\sigma^{-1}(\sigma(t))$ =t, $\sigma'(t) \in \mathrm{H}_\omega(\Gamma)$ qoida bilan akslantirish bajaruvchi chiziqli chegatalangan teskarilanuvchi operatorni belgilaymiz.

Teorema 1. Agar $(T \times T)\ da$ noldan farqli

$\frac{\sigma^{-1}(\xi)-\sigma^{-1}(z)}{\xi - z} \in \mathrm{H}_\omega(T \times T)$ berilgan funksiya bo'lsa, S_T uchun quyidagi tenglik o'rinli bo'ladi:

$$S_T = BS_\Gamma B^{-1} + K \qquad (6)$$

Bu yerda K $\mathrm{H}_\omega(T)$ fazoda aniqlangan to'la uzluksiz operatordir.

Isbot. Faraz qilaylik ψ∈$\mathrm{H}_\omega(T)$, φ=B^{-1} ψ va K = $S_T - BS_\Gamma B^{-1}$ bo'lsin. U holda

$(K\psi)(z) = \frac{1}{\pi i}\int_T \frac{\psi(\xi)}{\xi - z} d\xi - \frac{1}{\pi i}\int_T \frac{\varphi(\tau)}{\tau - \sigma^{-1}(z)} d\tau$ tenglik bajariladi. Ikkinchi integralda $\tau = \sigma^{-1}(\xi)$ almashtirish bajarib, quyidagi tenglikni hosil qilamiz:

$$(K\psi)(z) = \frac{1}{\pi i}\int_T \left(\frac{1}{\xi - z} - \frac{\sigma^{-1'}(\xi)}{\sigma^{-1}(\xi) - \sigma^{-1}(z)}\right)\psi(\xi)d\xi \qquad (7)$$

$b(\xi, z) = \frac{\sigma^{-1'}(\xi)(\xi - z)}{\sigma^{-1}(\xi) - \sigma^{-1}(z)}$ belgilash kiritib (7) tenglikni

$$(K\psi)(z) = \frac{1}{\pi i}\int_T \frac{b(\xi, z)}{\xi - z}\psi(\xi)d\xi \qquad (8)$$

ko'rinishga keltiramiz. [2] da (8) ko'rinishdagi operator $\mathrm{H}_\omega(T)$ da kompaktligi isbotlangan. Teorema isbot bo'ldi. Endi $\mathrm{H}_\omega(\Gamma)$ da WS-SW kommutatorni kompaktligini isbotlaymiz.

Teorema 2. Faraz qilaylik α-oddiy yopiq, silliq Γ chiziqni o'ziga akslantiruvchi N varaqli (N≥ 2) siljish funksiyasi bo'lib, $\alpha'(t) \in \mathrm{H}_\omega(\Gamma)$ shartlarni qanoatlantirsin va Γ ni yo'nalishini saqlasin. Agar α Γni yo'nalishini saqlasa, WS≃SW, agar α yo'nalishini o'zgartirsa, WS≃ −SW munosabatlar o'rinli bo'ladi. Bu yerda "≃ " belgi to'la uzluksiz operatorgacha aniqlikdagi tenglikni bildiradi.

Isbot. Faraz qilaylik σ Γ ni T birlik aylanaga o'tkazuvchi diffeomorfizm, $(\Gamma \times \Gamma)$ da noldan farqli $\frac{\sigma(t)-\sigma(\tau)}{t-\tau} \in \mathrm{H}_\omega(\Gamma \times \Gamma)$ funksiya berilgan bo'lsin.

U holda [7] maqolani 1-lemmasiga ko'ra T ni shunday β diffeomorfizmi topilib, β'∈$\mathrm{H}_\omega(T)$ shartda α akslantirishni α=$\sigma^{-1'} \circ \omega_{\pm N} \circ \beta \circ \sigma, \omega_{\pm N}(t) = t^{\pm N}, t \in T$ ko'rinishda tasvirlash mimkin. Bu yerda "+", "-" ishoralar α Γni yo'nalishini saqlashiga yoki o'zgartirishiga qarab tanlanadi. Osongina ko'rish mumkin $W_{\omega_{-N}}$=$W_C \cdot W_{\omega_N}$·$(W_C\varphi)$(t)= $\varphi(\bar{t})$. $\bar{t}$ t ga qo'shma kompleks son. Bunga ko'ra, [1] ni 35-betidagi 3.1. teoremasidan va mazkur maqolani 1-teoremasidan foydalanib, teoremani isbotlash uchun $W_{\omega_N}S \simeq S \cdot W_{\omega_N}$ munosabatni ko'rsatish yetarli.

Haqiqatan ham, faraz qilaylik α yo'nalishini saqlaydigan akslantirish bo'lsin (agar α yo'nalishini o'zgartirsa, fikrlash aynan takrorlanadi). $W_{\omega_N}S_T \simeq S_T \cdot W_{\omega_N}$tenglik bajarilganda [2] 4-xossasiga ko'ra $S_T W_{\beta^{-1}} \simeq W_{\beta^{-1}} \cdot S_T$ bajarilib,

$S_T W_{\beta^{-1}} \simeq W_{\beta^{-1}\omega_N^{-1}} \cdot S_T W_{\omega_N}$ va $S_T \simeq W_{\beta^{-1}\circ\omega_N^{-1}} \cdot S_T W_{\omega_N \circ \beta}$

munosabatlar o'rinli bo'ladi. Bundan mazkur maqolaning 1-teoremasiga ko'ra

$W_\sigma S_\Gamma W_{\sigma^{-1}} \simeq W_{\beta^{-1}\circ\omega_N^{-1}} \cdot S_T W_{\omega_N\circ\beta\circ\sigma} W_{\sigma^{-1}}$ bo'lib,

$$W_\sigma S_\Gamma \simeq W_{\beta^{-1}\circ\omega_N^{-1}} \cdot S_T W_{\omega_N\circ\beta\circ\sigma} \simeq W_{\beta^{-1}\circ\omega_N^{-1}} \cdot W_\sigma S_\Gamma W_{\sigma^{-1}} W_{\omega_N\circ\beta\circ\sigma} \Leftrightarrow$$

$$\Leftrightarrow W_{\beta^{-1}\circ\omega_N^{-1}\circ\sigma} \cdot W_{\sigma^{-1}\circ\omega_N\circ\beta\circ\sigma} S_\Gamma \simeq W_{\beta^{-1}\circ\omega_N^{-1} S_\Gamma \circ\sigma} W_{\sigma^{-1}\circ\omega_N\circ\beta\circ\sigma} \Leftrightarrow$$

$\Leftrightarrow W_{\sigma^{-1}\circ\omega_N\circ\beta\circ\sigma} S_\Gamma \simeq S_\Gamma W_{\sigma^{-1}\circ\omega_N\circ\beta\circ\sigma} \simeq W_\alpha S_\Gamma \simeq S_\Gamma W_\alpha$ munosabatni hosil qilamiz. Bunda foydalanilgan W_{ω_N} S_T-$S_T W_{\omega_N}$ kommutatorning kompaktligi isboti [3] maqolaning 7.1. lemmasiga o'xshash bo'lgani uchun keltirilmadi. Teorema isbot bo'ldi.

Shuni ta'kidlaymizki, agar $\omega(\delta)\sim\delta^\mu$, $(0< \mu<1)$ bo'lsa, oddiy Gyolder fazosi $H_\mu$da silliq chiziqning $\alpha'(t) \in H_\mu$ shartda chekli varaqli endomorfizmi uchun WS - SW kommutatorning kompaktligi [8] maqolaning 1.1. lemmasi va 1-teoremadan kelib chiqadi.

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