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BOUNDARY VALUE PROBLEM IN LOWER SEMIFIELDS DEVIATING FROM CHARACTERISTICS FOR A PARABOLIC–HYPERBOLIC EQUATION

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Annotation: This article proves the existence and uniqueness of a solution to a nonlocal conditional boundary value problem for a boundary value problem in lower semifields containing lines shifted from the characteristic for an equation of parabolic-hyperbolic type.

Key words: Equation of parabolic-hyperbolic type, characteristic triangle, usual solution, integral energy method, simple solution, Green's function, Volterra integral equation of the second kind, Dalembert's formula.

1. Setting the issue

Consider the following equation:

$$0 = Lu \equiv \begin{cases} u_{xx} - u_y, & (x,y) \in \Omega_0, \\ u_{xx} - u_{yy}, & (x,y) \in \Omega_j \left(j = \overline{1,3}\right), \end{cases} \tag{1}$$

here Ω_0 as a field $x > 0,\ y > 0$ when $y = 0,\quad x = 1,\quad y = 1,\quad x = 0$ located in straight lines $AB,\ BB_0,\ B_0A_0,\ A_0A,$ a rectangular area bounded by sections, Ω_1 field $x < 0,\ y > 0$ da ΔAA_0D is located inside the characteristic triangle $AK: x = \gamma_1(y)$ smooth curve and equation (1). $BP:\ y - x = 1$ area limited by characteristic, Ω_2 field in $x > 0,\ y > 0$ ΔBB_0E is located inside the characteristic triangle $AC: x = -\gamma_2(y)$ smooth curve and equation (1). $B_0M:\ x + y = 2$ area limited by characteristic, Ω_3 field in $x < 0,\ y < 0$ (1) of the equation $AC:\ y + x = 0$va $BC:\ y - x = -1$ limited by the characteristic $\Delta\, AA_0D$ characteristic triangular area,

Let us introduce the following definitions: $J_1 = \left\{(x,y):\ 0 < x < 1,\ \ y = 0\right\}$,

$J_2 = \left\{(x,y):\ x = 0, 0 < y < 1\right\}, J_3 = \left\{(x,y):\ x = 1,\ 0 < y < 1\right\},$

$$\Omega=\Omega_0\cup\Omega_1\cup\Omega_2\cup\Omega_3\cup J_1\cup J_2\cup J_3, C\left(-\frac{1}{2},\frac{1}{2}\right),\quad D\left(-\frac{1}{2},\frac{1}{2}\right),\quad E\left(\frac{3}{2},\frac{1}{2}\right),$$

$$\theta\left(\frac{x}{2};-\frac{x}{2}\right),\ \left[\theta^*\left(\frac{\lambda(x)+x}{2};\frac{\lambda(x)-x}{2}\right)\right]$$

there $\theta_2(x)\left[\theta_2^*(x)\right]$ (1) of Eq $(x,0)\in J_3$ with point-to-point characteristics $AC[AN]$ the coordinates of the intersection point of the characteristics.

A-Condition. $y=-\gamma_1(x)$va $\gamma_j(y)(j=2,3)$- let the given functions fulfill the following conditions:

1) $\gamma_1(x)$va $\gamma_j(y)(j=2,3)$ functions respectively $\Delta ACB$and $\Delta AA_0D, \Delta BEB_0$ should be completely inside the characteristic triangles;

2) $\gamma_1(x)\in C^2(0,1), \gamma_j(y)\in C^2(0,1)(j=2,3)$ be relevant;

3) $t\pm\gamma_j(t)\left(j=\overline{1,3}\right)$- monotonically growing;

4) $\gamma_1(0)=0,\ \gamma_2(0)=-1,\ l_1+\gamma_1(l_1)=1,\ l_2-\gamma_2(l_2)=2, l_3+\gamma_3(l_3)=1,\ l_j=const$ $l_j\in\left(\frac{1}{2};1\right)$.

Description. (1) as a regular solution of Eq Ω ,

$$W_1=\left\{u:u(x,y)\in C^1\left(\overline{\Omega}\right)\cap C^{2,1}_{x,y}\left(\Omega_0\right)\cap C^2\left(\Omega_i\right), i=\overline{1,3}\right\}$$

belonging to the class and $\Omega_i\left(i=\overline{0,3}\right)$in the field (1) satisfying Eq $u(x,y)$ the solution is told.

I – Matter. Find a regular solution of equation (1) satisfying the following conditions:

$$\left[u_x-u_y\right]\theta(x)+\mu(x)\left[u_x-u_y\right]\theta^*(x)=\varphi(x); \tag{2}$$

$$u|_{A_0D}=g_1(y);\ \ u|_{B_0E}=g_2(y) \tag{3}$$

$$\left[u_x+u_y\right]\Big|_{AD}=p(y); \tag{4}$$

$$\left[u_x-u_y\right]\Big|_{BE}=q(y); \tag{5}$$

$$u(A)=u(B)=0; \tag{6}$$

There $\mu(x),\ \varphi(x),\ g(y),\ p(y)$va $q(y)$- are sufficiently smooth functions given.

Theorem. If $\mu(x)\neq-1;\ \mu(x),\varphi(x)\in C^1[0,1]\left(i=\overline{1,3}\right), g(y),\ p(y),\quad q(y)\in C^2(0;1)$ and **A** the conditions are met, I-is the only regular of the problem there will be a solution.

Proof. Ω_1 The appearance of the solution of the Cauchy problem in fields is as follows.

$$u(x,y)=\frac{1}{2}\left[\tau_1(x+y)+\tau_1(x-y)+\int_{x-y}^{x+y}\nu_1(t)dt\right],\ 0<x<1$$

From this $\theta\left(\frac{x}{2};-\frac{x}{2}\right)$ and $\theta^*\left(\frac{\lambda(x)+x}{2};\frac{\lambda(x)-x}{2}\right)$ we get the following functional relations at the points:

$$(1+\mu(x))\nu_1(x)=[1+\mu(x)]\tau_1'(x)-\varphi(x),\ 0<x<1 \qquad (7)$$

From (1) Ω_0 in the field $y\to+0$ passing to the limit, we get the following equation.

$$\left(\tau_1(x)\right)''=\nu_1(x) \qquad (8)$$

Substituting (7) into (8). We can form a second-order ordinary differential equation with respect to $\tau_1(x)$.

$$(\tau_1(x))''-\tau_1'(x)=-\frac{\varphi(x)}{1+\mu(x)} \qquad (9)$$

(9) equation $\tau_1(0)=0$ and $\tau_1(1)=0$ the shape of the general appearance of the picture under the conditions.

$$\tau_1(x)=\int_0^x\frac{\varphi(t)\left[1-e^{x-t}\right]}{1+\mu(t)}dt+\frac{e^x-1}{e-1}\int_0^1\frac{\varphi(t)\left[e^{1-t}-1\right]}{1+\mu(t)}dt \qquad (10)$$

using (8), we find $\nu_1(x)$

$$\nu_1(x)=-\int_0^x\frac{\varphi(t)e^{x-t}}{1+\mu(t)}dt+\frac{e^x}{e-1}\int_0^1\frac{\varphi(t)\left(e^{1-t}-1\right)}{1+\mu(t)}dt-\frac{\varphi(x)}{1+\mu(x)} \qquad (11)$$

We prove the existence of a solution to the problem by the method of integral equations. For this, we use the functional relations (8) - (9) and the solution of the first boundary value problem for the equation (1) in domain Ω_0

$$u(x,y)=\int_0^1\tau_1(t)G(x,y;\text{t},0)dt+\int_0^y\tau_2(t)G_t(x,y;0,\text{z})dz-$$
$$-\int_0^y\tau_3(z)G_t(0,y;1,\text{z})dz, \qquad (12)$$

it will be visible.

There $G(x,y;t,z)=\frac{1}{2\sqrt{\pi(y-z)}}\sum_{n=-\infty}^{\infty}\left[e^{-\frac{(x-t+2n)}{4(y-z)}}-e^{-\frac{(x+t+2n)}{4(y-z)}}\right]$ - Green's function of the first boundary value problem for the heat transfer equation.

$\tau_k(y),\ \nu_k(y)\ (k=2,3)$ to get the relation between the functions, differentiate once: We make

$$u_x(x,y)=\int_0^1\tau_1(t)G_x(x,y;t,0)dt+\int_0^y\tau_2(z)G_{tx}(x,y;0,z)dz-$$
$$-\int_0^y\tau_3(z)G_{tx}(x,y;1,z)dz, \qquad (13)$$

The following $N(x,y;t,z)=\frac{1}{2\sqrt{\pi(y-z)}}\sum_{n=-\infty}^{\infty}\left[e^{-\frac{(x-t+2n)^2}{4(y-z)}}+e^{-\frac{(x+t+2n)^2}{4(y-z)}}\right]$ if we define,

$G_{tx}(x,y;t,z)=N_z(x,y;t,z),\ G_x(x,y;t,z)=-N_t(x,y;t,z)$ We will have relationships. According to definition, $u_x(0,y)=\nu_2(y)$ we get the following relations from (13):

$$\nu_2(y)=\int_0^1\tau_3'(z)N(x,y;1,z)dt-\int_0^y\tau_1'(t)N(x,y;t,0)dt+\int_0^y\tau_2'(z)N(0,y;0,z)dz$$
$$\nu_3(y)=\int_0^1\tau_3'(z)N(1,y;1,z)dz-\int_0^y\tau_1'(z)N(1,y;t,0)dz+\int_0^y\tau_2'(z)N(1,y;0,z)dz \qquad (14)$$

It is known, The general solution of equation $u_{xx}-u_{yy}=0$ is in the form

$$u(x,y)=f_1(x+y)+f_2(x-y) \qquad (15)$$

in this $f_1(.),f_2(.)$ - second-order continuous differentiation is an unknown function.

From condition (4) and (15), we get $f_1'(y-\gamma_2(y))=p(y),\ 0\le y\le l$, from Eq $y-\gamma_2(y)=t$ solve looking for $y=\delta_1(t)$

$$f_1'(t)=\frac{1}{2}p(\delta_1(t)),\ 0\le y\le l, \text{ From this}$$

$$f_1(y)=f_1(0)+\frac{1}{2}\int_0^y p(\delta(t))dt,\ 0\le y\le l.$$

From condition (3.8) and from (15). we will have 1 $f_2'(y-\gamma_2(y))=q(y),\ 0\le y\le l$, from Eq solve $y-\gamma_2(y)=t$ looking for $y=\delta_1(t)$

$$f_2'(t)=\frac{1}{2}q(\delta_1(t)),\ 0\le y\le l, \text{ From this}$$

$$f_2(y)=f_2(0)+\frac{1}{2}\int_0^y q(\delta(t))dt,\ 0\le y\le l.$$

Now in $l\le y\le 1$ $u|_{A_0D}=g_1(y)$ and given $u|_{B_0E}=g_2(y)$ conditions

$$\begin{cases} f_1(y)=g_1\left(\frac{y-1}{2}\right)+f_2(1),\ l\le y\le 1 \\ f_2(y)=g_2\left(\frac{2-y}{2}\right)+f_1(2),\ l\le y\le 1 \end{cases}$$

To (15) we put the value of $f_1(y)$and $f_2(y)$ and get the following.

$$u(x,y)=\begin{cases} f_2(x-y)+\frac{1}{2}\int_0^y p(\delta(t))dt+f_1(0),0\le y\le l, \\ f_2(x-y)+g_1\left(\frac{y-1}{2}\right)+f_2(1),\ l\le y\le 1. \\ f_1(x+y)+\frac{1}{2}\int_0^y q(\delta(t))dt+f_2(0),0\le y\le l, \\ f_1(x+y)+g_2\left(\frac{2-y}{2}\right)-f_1(2),\ l\le y\le 1. \end{cases} \tag{16}$$

$u_y(0,y)=\tau_i'(y),\ i=2,3,$ If we differentiate the equality (16) once with respect to *y*, we say $x\to 0$

$$\tau_2'(y)=\begin{cases} -f_2'(-y)+\frac{1}{2}p(\delta(y)), & 0\le y\le l, \\ -f_2'(-y)+\frac{1}{2}g_1'\left(\frac{y-1}{2}\right), & l\le y\le 1. \end{cases} \tag{17}$$

And at $x\to 1$

$$\tau_3'(y)=\begin{cases} f_1'(1+y)+\frac{1}{2}q(\delta(1+y)), & 0\le y\le l, \\ f_1'(1+y)-\frac{1}{2}g_2'\left(\frac{1-y}{2}\right), & l\le y\le 1. \end{cases} \tag{18}$$

Putting (16) into (17) and (18). $f_i'(y)$ $i=2,3$ for the function $\tau_i(y)$ $i=2,3$ and $\nu_i(y)$ $i=2,3$ we get the following functional relation between the functions:

$$\begin{cases} \tau_2'(y) = \nu_2(y) + p(\delta(y)), & 0 \le y \le l,\ x < 0 \\ \tau_2'(y) = \nu_2(y) + g_1'\left(\frac{y-1}{2}\right), & l \le y \le 1.\ x < 0 \end{cases} \quad (19)$$

$$\begin{cases} \tau_3'(y) = \nu_3(y) + q(\delta(1-y)), & 0 \le y \le l,\ x > 1 \\ \tau_3'(y) = \nu_3(y) + g_2'\left(\frac{1-y}{2}\right), & l \le y \le 1, \quad x > 1 \end{cases} \quad (20)$$

From (19) and (20). $\nu_2(y)$ and If we put $\nu_3(y)$ in (14)

$$\tau_2'(y) - \int_0^1 \tau_3'(z) N(x, y; 1, z)dt + \int_0^y \tau_1'(t) N(x, y; t, 0)dt - \int_0^y \tau_2'(z) N(0, y; 0, z)dz = F_2(y)$$

$$\tau_3'(y) - \int_0^1 \tau_3'(z) N(1, y; 1, z)dz + \int_0^y \tau_1'(z) N(1, y; t, 0)dz - \int_0^y \tau_2'(z) N(1, y; 0, z)dz = F_3(y)$$

(21)

There

$$F_2(y) = \frac{1}{2} p(\delta(1-y)) - \frac{1}{2} g_1'\left(\frac{1-y}{2}\right)$$

$$F_3(y) = \frac{1}{2} q(\delta(1-y)) - \frac{1}{2} g_2'\left(\frac{1-y}{2}\right)$$

We get the system (21) in the form

$$\begin{cases} \tau_2'(y) - \int_0^y \tau_2'(z) N(0, y; 0, z)dz = F_2^*(y), \\ \tau_3'(y) + \int_0^y \tau_3'(z) N(1, y; 1, z)dz = F_3^*(y). \end{cases} \quad (22)$$

Where

$$\begin{cases} F_2^*(y) = F_2(y) + \int_0^y \tau_3'(z) N(x, y; 1, z)dz - \int_0^1 \tau_1'(t) N(x, y, t, 0)dt, \\ F_3^*(y) = F_3(y) + \int_0^y \tau_2'(z) N(1, y; 0, z)dz - \int_0^y \tau_1'(z) N(1, y, t, 0)dt \end{cases} \quad (23)$$

(14) system $\left|N(0, y, 0, z)\right| \le \frac{2}{\sqrt{\pi|y - y_1|}} \sum_{n=1}^{\infty} \left| e^{-\frac{n^2}{|y-y_1|}} \right| \le const$

$\left|F_2^*(y)\right| \le const$ because we solve equation 1 from the system (23) by the method of successive approximation and get

$$\tau_2(y) = F_2^*(y) + \int_0^y F_2^*(t) K(t, y)dt \quad (24)$$

putting $F_2^*(y)$ in (24).

$$\tau_2(y)=F_2(y)+\int_0^y \tau_3'(z)N(x,y;1,z)dz-\int_0^1 \tau_1'(t)N(x,y,t,0)dt+$$
$$+\int_0^y\left(F_2(y)+\int_0^t \tau_3'(z)N(x,y;1,z)dz-\int_0^y\int_0^1 \tau_1'(p)N(x,y,p,0)dp\right)K(t,y)dt \qquad (25)$$

And finally, we put (25) into equation 2 of (21) and $\tau_2(y)$, we form the Volterra integral equation of the second type:

$$\tau_3'(y)-\int_0^y \tau_3'(z)N(1,y;1,z)dz+\int_0^y \tau_1'(z)N(1,y;t,0)dz-\int_0^z N(1,y;0,z)dz-\int_0^y \tau_3'(z)N(x,y;1,z)+$$
$$+\int_0^y \tau_3'(z)N(x,y;1,z)dz-\int_0^y\int_0^1 \tau_1'(p)N(x,y,p,0)dpK(t,y)dp+\widetilde{F_3}(y)=F_2(y)$$

(26)

There

$$\widetilde{F_3}(y)=\int_0^y F_2(z)N(1,y,0,z)dz+\int_0^y F_2(s)K(s,y)ds+\int_0^y N(1,y,0,z)\int_0^y F_2(p)K(p,y)dp$$

(27)

(26) tenglamani yechib $\tau_2(y)$, bundan va (25) dan $\tau_1(y)$va (26), (27)dan $\nu_1(y)$va $\nu_2(y)$ni topamiz. $\tau_i(y),\nu_i(y)\ \left(i=\overline{1,3}\right)$ lar ma'lum funksiyalar. Endi Ω_0 sohada I masalaning we can restore the solution, $\overline{\Omega_i}\left(i=\overline{1,3}\right)$in fields, the solution is found through the Dalamber's formula, which is the solution of the Cauchy problem. So, I-problem is solved one-valued.

The theorem is proved.

References

1. Salohiddinov M. Matematik – fizika tenglamalari. “O’zbekiston” nashriyoti. T. 2002 y.
2. Рахматуллаева Н. А Локальные и нелокальные задачи для параболо – гиперболических уравнений с тремя линиями изменения типа. //канд. диссертация. Ташкент. 2011. 96. стр.
3. Шоимов Б. С Единственность решение нелокальной задачи для уравнения параболо – гиперболического типа в области с отходом от характеристики . // “Актуальные вопросы анализа” Қарши. 2016. С. 198 – 200.
4. Михлин С.Г. Лекции по линейным интегральным уравнениям. М.: Физматгиз. 1959. 232 с.
5. Shoimov. B.S. Parabolik–giperbolik tipdagi tenglamalar uchun xarakteristikadan siljigan chiziqlarni o`z ichiga olgan sohalarda chegaraviy masala.// Namangan Davlat Universitetining ilmiy axborotnomasi 2022-yil 6-son [48-54].
6. Shoimov B.S, Jamolov Sh. Singulyar koeffitsientga ega bo’lgan giperbolik tipdagi tenglama uchun koshi masalasi.// Buxoro Davlat Universiteti ilmiy axborotnomasi 2023-yil 2-son [65- 70].
7. Федеpatsiи. Ж, “Молочное и мясное скотоводство”, №6, 2010, с.2-5.