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Calculation of Differential Equations

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This paper provides an overview of differential equations and provides solutions to some examples. We have considered and solved the problem of teaching a single-layer neural network belonging to the field of artificial intelligence of programming using differential equations, special derivatives. We programmed and solved some differential problems and created a graph of the differential equation. An arbitrary

$$f(x, y, y', y'', \dots, y^{(n)})=0$$

The relationship is called a simple differential equation. The highest order of the products included in the differential equation is called the order of the differential equation. The solution of a differential equation is the differentiable function y=φ(x), which, when applied to the equation, becomes an identity. For such an equation, the Cauchy problem is called the initial condition

$$y|_{x=xo}=y_0,\ y'|_{x=xo}= y'_0,\ \dots,\ y^{(n-1)}|_{x=xo}=y_o^{(n-1)}$$

is to find a solution that satisfies the conditions.

$$M(x)dx+N(y)dy = 0$$

is called a differential equation in which the variables of the form are separated. Its peculiarity is that dx is preceded only by a multiplier dependent on x, and dy is preceded only by a multiplier dependent on y. The solution of this equation is determined by its gradual integration:

$$\int M(x)dx + \int N(y)dy = C.$$

An implicit solution to a differential equation is called an integral of the equation. The integration constant C can be chosen in a way that is convenient for the solution. Example 1. tgxdx-ctgxdy = 0 Find the general solution of the equation.

Solution. Bu yerda o'zgaruvchilari ajralgan tenglamaga egamiz. Uni hadma-had integrallaymiz:

$$\int tgxdx - \int ctgxdy = C \qquad va \qquad -ln|cosx| - ln|siny| = ln|C|$$

$$-ln|cosx \cdot siny| = ln|C|, \qquad ln\left|\frac{1}{cosx \cdot siny}\right| = ln|C|, \qquad |C| = \left|\frac{1}{cosx \cdot siny}\right|$$ bu yerda

$|C| = \left|\frac{1}{cosx \cdot siny}\right|$ tenglamaning integrali. Umumiy yechimni topish uchun quyidagi hollarni qaraymiz:

1) C>0, $\frac{1}{cosx \cdot siny}$>0 va C<0, $\frac{1}{cosx \cdot siny}$<0 bunda $|C| = \left|\frac{1}{cosx \cdot siny}\right|$ tenglamadagi modullarni tashlab yuboramiz, ya'ni $C = \frac{1}{cosx \cdot siny}$;

2) C<0, $\frac{1}{cosx \cdot siny}$>0 va C>0, $\frac{1}{cosx \cdot siny}$<0 bunda esa $C = -\frac{1}{cosx \cdot siny}$ deb olish mumkin;

Demak, ikki fikrimizni birlashtirsak, $C = \pm\frac{1}{cosx \cdot siny}$ tenglamaga kelamiz. Endi umumiy yechimni quyidagicha yozish mumkin: y= $\pm$arcsin $\left(\frac{1}{C \cdot cosx}\right)$.

$f(x, y(x), y'(x))=f(x, y(x), \frac{dy}{dx})$ ko'rinishdagi tenglamaga birinchi tartibli differensial tenglamalar deyiladi. Bu yerda x− erkli o'zgaruvchi, y= y(x) – noma'lum funksiya.

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Bularga misol qilib, $x^2 - 2 + y + 6y' = 0$, $\sqrt{xy} - 1 - 2y' = 0$ larni keltirish mumkin. Birinchi tartibli hosilaga nisbatan yechilgan differensial tenglama deb,

$$\frac{dy}{dx} = f(x, y) \qquad \text{yoki} \qquad M(x, y)dx+N(x, y)dy=0$$

ko'rinishdagi tenglamalarga aytiladi. Bu yerda f(x, y), M(x, y), N(x, y) lar berilgan funksiyalar. Masalan: $\frac{dy}{dx} = \sin x \cdot \cos y$, $\sqrt{x \cdot y} - 5 + y' = 0$.

Agar differensial tenglamadagi noma'lum funksiya ikki va undan ortiq ko'p argumentlarga bog'liq bo'lsa, u xususiy hosilali differensial tenglama deyiladi. Bunday tenglamalarning nomidan ko'rinib turubdiki, ularda funksiyaning erkli argumentlari bo'yicha xususiy hosilalari qatnashadi. Oddiy differensial tenglamalardagi kabi xususiy hosilali differensialtenglamalar ham cheksiz ko'p yechimlarga ega. Bu yechimlarga umumiy yechimlar deyiladi. Xususiy yechimlar umumiy yechimlardan ma'lum shartlar asosida ajratiladi. Bu qo'shimcha shartlar tenglama qaralayotgan sohaning odatda chegarasida beriladi.

Agar

P(x,y)dx+Q(x,y)dy=0

Tenglamaning chap tomonini birorta U(x,y) funksiyaning to'liq differensiali, ya'ni

$$P(x,y)dx+Q(x,y)dy=dU(x,y)$$

bo'lsa, tenglama to'liq differensialli tenglama deyiladi. Bu holda uni dU(x, y)=0 ko'rinishda yozish mumkin va bu yerdan U(x, y)=C umumiy integralga ega bo'lamiz. Bu yerda P(x, y) va Q(x, y) funksiyalar D sohada aniqlangan va uzluksiz bo'lib, uzluksiz $\frac{\partial P(x,y)}{\partial y}, \frac{\partial P(x,y)}{\partial x}$ xususiy hosilalarga ega bo'lishi talab qilinadi. Misol. To'liq differensialli tenglamani yeching: $\frac{1}{x}dy - \frac{y}{x^2}dx = 0$; Yechish. $\frac{dy}{x} - \frac{y}{x^2}dx = 0$ tenglamaning chap qismi U= $\frac{y}{x}$ funksiyaning to'liq differensiali ekanligini ko'rish oson. Shuning uchun tenglamani $d(\frac{y}{x}) = 0$ ko'rinishda qayta yozib olamiz, bu yerdan y = Cx umumiy yechimga ega bo'lamiz.

n-tartibli differensial tenglamani simvolik ravishda

$$F(x, y, y', y'', \dots, y^{(n)})=0$$

ko'rinishda yoki bu tenglamani n-tartibli hosilaga nisbatan yechib bo'lsa,

$$y^{(n)} = f(x, y, y', y'', \dots, y^{(n-1)})$$

ko'rinishda yozish mumkin. n-tartibli differensial tenglamaning umumiy yechimi x ga va n-ta ixtiyoriy o'zgarmaslarga bog'liq bo'ladi:

$$y= g(x, C_1, C_2, C_3, \dots, C_n).$$

Misol. y=sin(x+C), $-\infty < x < +\infty$, $-\infty < C < +\infty$ chiziqlar oilasining differensial tenglamasi topilsin. $\begin{cases} y' = \cos(x + C), \\ y = \sin(x + C) \end{cases}$ munosabatlardan $y'^2+y^2=1$, $-\infty < x < +\infty$ differensial tenglama kelib chiqadi.

Differensial tenglamalarni o'rganish asosan ularning yechimlarini (har bir tenglamani qondiradigan funksiyalar to'plami) va ularning yechimlari xususiyatlarini o'rganishdan iborat. Faqat eng sodda differensial tenglamalar aniq formulalar bilan yechilishi mumkin; ammo, berilgan differensial tenglama yechimlarining ko'pgina xossalari ularni to'liq hisoblamasdan aniqlanishi mumkin.

Differensial tenglamalardan, xususiy hosilali differensial tenglamalardan deyarli barcha sohalarda foydalaniladi. Xususan, axborot texnologiyalari (dasturlash)da ham differensial tenglamalar, xususiy hosilalardan foydalangan holda turli masala va muammolarning yechimi topiladi. Neyron tarmoqlari misollarni qayta ishlash orqali o'rganadi (yoki o'qitiladi), ularning har biri ma'lum bo'lgan "kirish" va "natija" ni o'z ichiga oladi, bu ikkalasi o'rtasida aniqlik tarkibidagi ma'lumotlar assotsiatsiyasini shakllantiradi. Nerv tarmog'ini keltirilgan misoldan o'rgatish odatda tarmoqning qayta ishlangan chiqishi (ko'pincha bashorat qilish) va maqsadli chiqish o'rtasidagi farqni aniqlash orqali amalga oshiriladi. Bu xato. Keyin tarmoq o'z qoidalariga binoan va ushbu xatolik qiymatidan foydalangan holda o'z vaznli assotsiatsiyalarini sozlaydi. Ketma-ket tuzatishlar neyron tarmoqni maqsadli chiqishga tobora ko'proq o'xshash ishlab chiqarishni keltirib chiqaradi. Ushbu tuzatishlarning yetarli sonidan so'ng ma'lum me’zonlarga asoslanib, mashg'ulot tugatilishi mumkin. Endi fikrimiz tasdig’i sifatida quyidagi dasturni tuzib natijasini olamiz:

Masalaning qo’yilishi: Bir qatlamli neyron to’rini o’qitish: x=[0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, 2.7, 3.0] va y=[1.3, 1.6, 1.9, 2.2, 2.5, 2.8, 3.1, 3.4, 3.7, 4.0] massivlar berilgan. Ushbu neyron tarmoqning takroriy o’qitilishlari soni (epoch), o’rtacha kvadratik xatoligi (MSE), yaxshilangan bashorat vazni (weight ya’ni dasturimiz uchun w), yaxshilangan, bashorat qilingan bias (b) lar aniqlansin va grafik orqali tasvirlansin.

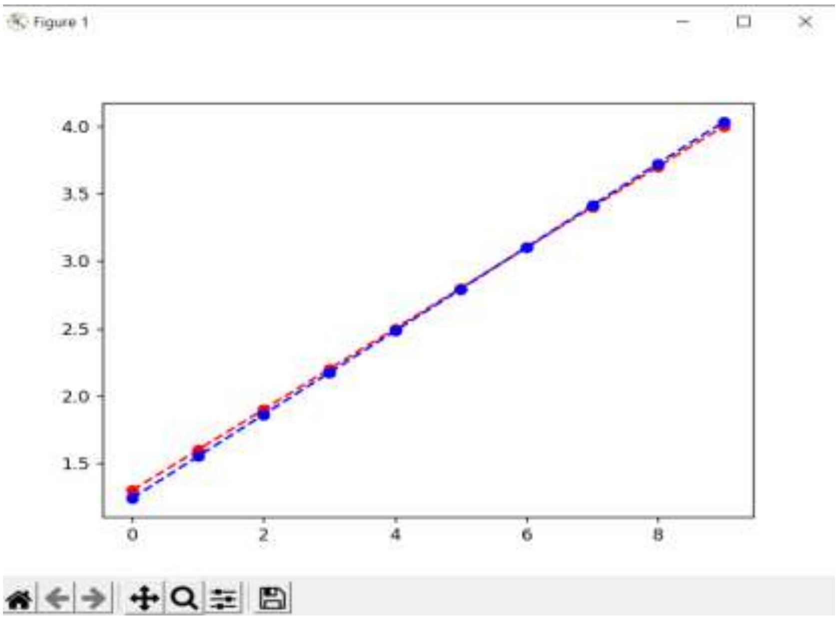
Pythondagi dasturi quyidagicha bo’ladi:

```
import matplotlib.pyplot as plt
 x=[0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, 2.7, 3.0]
 y=[1.3, 1.6, 1.9, 2.2, 2.5, 2.8, 3.1, 3.4, 3.7, 4.0]
 w=0
 b=0
alfa = 0.001
MSE_min = 0.001
 for epoch in range(10000):
 s = 0
 for x_in, y_in in zip(x,y):
 y_pred=x_in*w+b
 E_error = (y_pred-y_in)**2
 grad_w = 2*(y_pred - y_in)*x_in
 w = w - alfa* grad_w
 grad_b= 2*(y_pred - y_in)
 b = b- alfa * grad_b
 s = s+E_error
 if(s/len(x)<MSE_min):
 break
 print("epoch", epoch)
 MSE=s/len(y)
 print("MSE=", MSE)
 print("w=",w)
 print("b=",b)
y_pArr = []
 for x_input in x:
 y_input = x_input*w+b
 y_pArr.append(y_input)
# plt.plot(y.color=="red")
plt.plot(y,'o--r')
plt.plot(y_pArr,'o--b')
plt.show()
```

Natija olish:

```
epoch 506
MSE= 0.0010076010011444208
w= 1.0337380679728108
b= 0.9319432340573282
epoch 507
MSE= 0.0010006321996727514
w= 1.0336211954037535
b= 0.9321789905648994
```

Grafigi:



Agar v(t) tezlik ma'lum bo'lsa, s(t) yo'lni topish masalasi s'(t)=v(t) differensial tenglamani yechishga keladi. Masalan, v(t)=8t-5 bo'lsa, u holda s(t) ni topish masalasi s'(t)= 8t − 5 differensial tenglamani yechishga keltiriladi. Boshlang'ich olsak, $s(t)=4t^2-5t+C$ tenglama yechim bo'ladi. Umuman, fizika, texnika, biologiya, kimyo, tibbiyot, iqtisodiyot va dasturlash sohalarining ko'pgina amaliy masalalari y'(t)=k-y(t) differensial tenglamani qanoatlantiruvchi y(t) funksiyani topishga keladi, bu yerda k− berilgan biror o'zgarmas son. Bu tenglamaning yechimlari esa $y(t)=Ce^{kx}$ ko'rinishdagi har qanday funksiyadan iborat ekanligini ko'rish qiyin emas.

Misol: Umumiy markazi (a; b) nuqtada bo'lgan aylanalardan iborat bo'lgan egri chiziqlar oilasi differensial tenglamasidagi dy/dx ning aniq sondagi qiymatini topish dasturini tuzing. **Pythondagi dasturi** quyidagicha:

```
a=int(input("a ni kiriting= "))
b=int(input("b ni kiriting= "))
x=int(input("x ni kiriting= "))
y=int(input("y ni kiriting= "))
R=((x-a)2+(y-b)**2)(1/2)
# x bo'yicha differensiallasak, 2 ga bo'lsak
# (y-b)*z+(x-a)=0
# z=dy/dx
z=-(x-a)/(y-b)
print("dy/dx differensialning qiymati quyidagicha = ", z)
```

Natija olish:

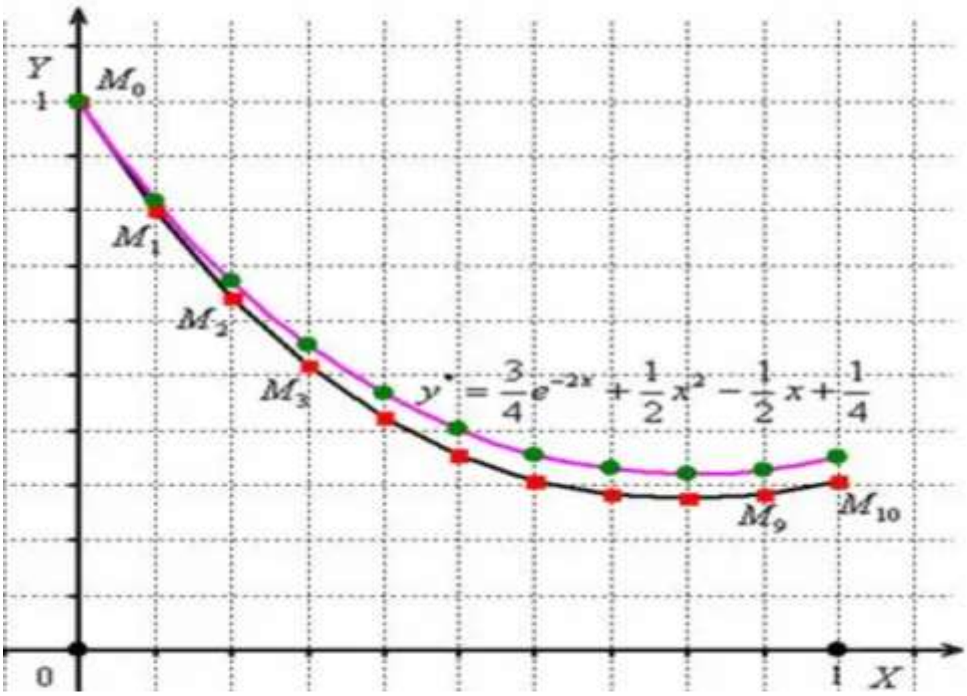
```
a ni kiriting= 1
b ni kiriting= 2
x ni kiriting= 4
y ni kiriting= 3
dy/dx differensialning qiymati quyidagicha =  -3.0
```

1-teorema. Agar funksiya nuqtaning biror atrofida aniqlangan, uzluksiz va uzluksiz xususiy hosilaga ega bo‘lsa, u holda nuqtaning shunday atrofi mavjudki, bu atrofda differensial tenglama uchun boshlang‘ich shartli Koshi masalasi yechimi mavjud va yagonadir. Differensial tenglamaning umumiy va xususiy yechimlari tushunchalariga aniqlik kiritamiz. Agar boshlang‘ich nuqtaning berilishi (2) tenglama yechimining yagonaligini aniqlasa, u holda ushbu yagona yechim xususiy yechim deyiladi. Differensial tenglamaning barcha xususiy yechimlari to‘plamiga uning umumiy yechimi deyiladi. Odatda, umumiy yechim oshkor yoki oshkormas ko‘rinishda yoziladi. o‘zgarmas boshlang‘ich shart asosida tenglamadan topiladi. **Ta’rif.** Tenglamaning umumiy integrali (yoki yechimi) deb, o‘zgarmasning turli qiymatlarida barcha xususiy yechimlari aniqlanadigan munosabatga aytiladi.

Differensial tenglamani shartlarsiz yechish uning umumiy yechimini (yoki umumiy integralini) topishni anglatadi.

Differensial tenglamalar fani turli xil fizik jarayonlarni o'rganish bilan chambarchas bog’liqdir. Bunday jarayonlar qatoriga gidrodinamika, elektrodinamika masalalari va boshqa ko’plab masalalarni keltirish mumkin. Turli jarayonlarni ifodalovchi matematik masalalar ko’pgina umumiylikka ega bo’lib, differensial tenglamalar fanining asosini tashkil etadi.

Misol. Qadamli segmentda Eyler usulidan foydalanib, boshlang'ich shartga mos keladigan differensial tenglamaning grafigini tuzing. **Grafik:**



Conclusion

Xulosa qilib shuni lozimki, differensial tenglamalar katta va muhim tushunchalar hisoblanadi. Ushbu maqolada differensial tenglamani o’rganuvchilar, oliy matematikaga qiziquvchilar va talabalar uchun muhim bo’lgan tushunchalar keltirildi. Shu maqolani yozishim davomida mavzu bo’yicha ko’p narsalarni o’rgandim, dasturlash sohasiga tadbiqi, grafiklar tahlili haqida ma’lumotlar keltirildi.

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