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TOR TEBRANISH TENGLAMASI UCHUN CHEGARAVIY MASALADA KASR TARTIBLI DIFFERENSIAL OPERATOR QATNASHGAN SILJISHLI MASALA

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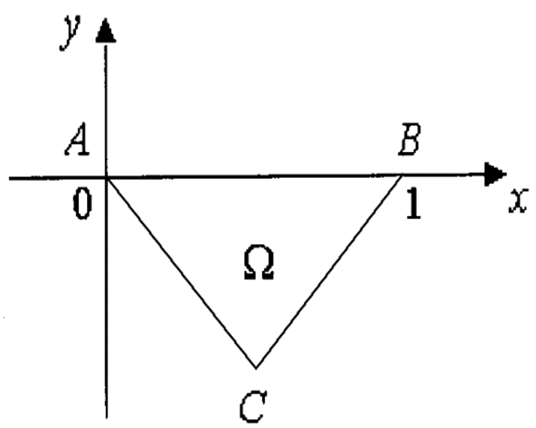
Annotatsiya : *Ushbu maqolada tor tebranish tenglamasi uchun chegaraviy masalada kasr tartibli differensial operator qatnashgan siljishli masalani yechimining mavjudligi va yagonaligi batafsil o'rganilgan.*

Kalit so'zlar: *Kasr tartibli differensial operator, regulyar yechim, giperbolik tipdagi tenglama, Koshi masalasi, yechimning mavjudligi, Volterra integral tenglamasi, yechimning yagonaligi.*

(x,y) -o'zgaruvchilar tekisligida

$$U_{xx} - U_{yy} = 0 \qquad (1)$$

tenglamaning $AC: x+y=0,\ BC: x-y=1$ xarakteristikalari va $y=0$ to'g'ri chiziqning



$\overline{J} = [0,1]$ kesmasi bilan chegaralangan Ω sohani ko'raylik.

Masala. Quyidagi shartlarni qanoatlantiruvchi $U(x,y)$ funksiya topilsin.

1) $U(x,y) \in C(\overline{\Omega}) \cap C^2(\Omega)$,

2) Ω sohada (1) tenglamani qanoatlantiradi.

3) Ω soha chegarasida

$$U(x,0)=\tau(x),\quad x\in\overline{J} \tag{2}$$

$$a(x)D_{0x}^{\alpha}U\left(\frac{x}{2},-\frac{x}{2}\right)+b(x)D_{x1}^{\beta}U\left(\frac{x+1}{2},\frac{x-1}{2}\right)=p(x)U_y(x,0)+q(x);\quad x\in J \tag{3}$$

Bu yerda $D_{0x}^{\alpha}f(y)=\dfrac{1}{\Gamma(1-\lambda)}\dfrac{d}{dy}\int\limits_0^x\dfrac{f(t)dt}{(x-t)^{\alpha}},\quad 0<\alpha<1,$ - Riman – Liuvill ma'nosidagi kasr tartibli differensial operator.

shartlarni qanoatlantiradi. Bu yerda $\alpha,\beta-$haqiqiy sonlar; $\tau(x)$, $a(x)$, $b(x)$, $p(x)$, $q(x)$-berilgan funksiyalar, D_{0x}^{α}, D_{x1}^{β} -kasr tartibli differentsial operatorlar [1].

1-teorema. Agar $U(x,y)$ funksiya (1)-(3) masalaning regulyar yechimi bo'lsa, u holda $U_y(x,0)=\nu(x)$ funksiya

$$2p(x)\nu(x)+a(x)D_{0x}^{\alpha-1}\nu(x)+b(x)D_{x1}^{\beta-1}\nu(x)=\gamma(x), \tag{4}$$

tenglik bilan topiladi.

Bu yerda

$$\gamma(x)=a(x)D_{0x}^{\alpha}\left[\tau(0)+\tau(x)\right]+b(x)D_{x1}^{\beta}\left[\tau(x)+\tau(1)\right]-2q(x) \tag{5}$$

Isboti. Giperbolik tipdagi tenglama uchun Koshi masalasi korrekt qo'yilganligiga asoslanib, masala yechimini

$$U(x,y)=\frac{1}{2}\left[\tau(x-y)+\tau(x+y)\right]+\frac{1}{2}\int\limits_{x-y}^{x+y}\nu(t)dt, \tag{6}$$

ko'rinishda qidiramiz, bu yerda $\nu(x)\in C^2(J)$ - noma'lum funksiya[2].

$U(x,y)$ funksiya (1) tenglama va (2) shartni qanoatlantiradi. Biz $\nu(x)$ funksiyani shunday tanlaymizki, (6) funksiya (3) shartni ham qanoatlantirsin.

Berilgan (6) formulaga asosan,

$$U\left(\frac{x}{2},-\frac{x}{2}\right)=\frac{1}{2}\left[\tau(x)+\tau(0)\right]-\frac{1}{2}\int\limits_0^x\nu(t)dt=\frac{1}{2}\left[\tau(x)+\tau(0)\right]-\frac{1}{2}D_{0x}^{-1}\nu(x), \tag{7}$$

$$U\left(\frac{x+1}{2},-\frac{x-1}{2}\right)=\frac{1}{2}\left[\tau(1)+\tau(x)\right]-\frac{1}{2}\int\limits_x^1\nu(t)dt=\frac{1}{2}\left[\tau(1)+\tau(x)\right]-\frac{1}{2}D_{x1}^{-1}\nu(x). \tag{8}$$

Bu tengliklarni (3) shartga qo'yib, (4) tenglikni hosil qilamiz.

$$a(x)D_{0x}^{\alpha}\left[\frac{1}{2}(\tau(0)+\tau(x))-\frac{1}{2}D_{0x}^{-1}\nu(x)\right]+b(x)D_{x1}^{\beta}\left[(\tau(x)+\tau(1))-\frac{1}{2}D_{x1}^{-1}\nu(x)\right]=$$

$$=\frac{1}{2}a(x)D_{0x}^{\alpha}(\tau(0)+\tau(1))-\frac{1}{2}a(x)D_{0x}^{\alpha-1}\nu(x)+\frac{1}{2}b(x)D_{x1}^{\beta}(\tau(x)+\tau(1))-$$

$$-\frac{1}{2}b(x)D_{x1}^{\beta-1}\nu(x)=p(x)\nu(x)+q(x)$$

$$2p(x)\nu(x)+a(x)D_{0x}^{\alpha-1}\nu(x)+b(x)D_{x1}^{\beta-1}\nu(x)=\gamma(x), \tag{4}$$

$$\gamma(x)=a(x)D_{0x}^{\alpha}\left[\tau(0)+\tau(x)\right]+b(x)D_{x1}^{\beta}\left[\tau(x)+\tau(1)\right]-2q(x) \tag{5}$$

Teorema isbotlandi.

1-hol: $\alpha<1,\quad \beta=1$ bo'lsin.

2-teorema. Agar $\tau(x)\in C^1(\overline{AB})\cap C^3(AB)$, $a(x)$, $b(x)$, $p(x)$, $q(x)\in C(\overline{AB})$, $2p(x)+b(x)\neq 0$ bo'lsa. U holda (1) – (3) masalaning yechimi mavjud va yagona bo'ladi.
Isbot. Bu holda (4) tenglikni quyidagi ko'rinishda yozish mumkin.

$$2p(x)\nu(x)+a(x)D_{0x}^{\alpha-1}\nu(x)+b(x)\nu(x)=\gamma(x), \quad (9)$$

$$[2p(x)+b(x)]\nu(x)+\frac{a(x)}{\Gamma(1-\alpha)}\int_0^x (x-t)^{-\alpha}\nu(t)dt=\gamma(x), \quad (10)$$

Integral operator ta'rifidan

$$\nu(x)+\int_0^x K_1(x,t)\nu(t)dt=\gamma_1(x), \quad 0<x<1 \quad (11)$$

tenglik kelib chiqadi.
Bu yerda

$$K_1(x,t)=\frac{a(x)(x-t)^{-\alpha}}{\Gamma(1-\alpha)(2p(x)+b(x))}, \qquad \gamma_1(x)=\frac{\gamma(x)}{2p(x)+b(x)}, \quad (12)$$

Hosil bo'lgan (11) tenglama ikkinchi tur Volterra integral tenglamasi bo'lganligi uchun (1)-(3) masala yechimi mavjud va yagonadir.

2-hol: $\alpha=1,\ \beta<1$ bo'lsin.

3-teorema. Agar $\tau(x)\in C^1(\overline{AB})\cap C^3(AB)$, $a(x)$, $b(x)$, $p(x)$, $q(x)\in C(\overline{AB})$, $2p(x)+a(x)\neq 0$ bo'lsa. U holda (1) – (3) masalaning yechimi mavjud va yagona bo'ladi.
Isbot. Bu holda (4) tenglikni quyidagi ko'rinishda yozish mumkin.

$$2p(x)\nu(x)+b(x)D_{x1}^{\beta-1}\nu(x)+a(x)\nu(x)=\gamma(x), \quad (13)$$

$$[2p(x)+a(x)]\nu(x)+\frac{b(x)}{\Gamma(1-\beta)}\int_x^1 (t-x)^{-\beta}\nu(t)dt=\gamma(x), \quad (14)$$

Integral operator ta'rifidan

$$\nu(x)+\int_x^1 K_2(x,t)\nu(t)dt=\gamma_2(x), \quad 0<x<1 \quad (15)$$

tenglik kelib chiqadi. Bu yerda

$$K_2(x,t)=\frac{b(x)(t-x)^{-\beta}}{\Gamma(1-\beta)(2p(x)+a(x))}, \qquad \gamma_2(x)=\frac{\gamma(x)}{2p(x)+a(x)}, \quad (16)$$

Hosil bo'lgan (15) tenglama ikkinchi tur Volterra integral tenglamasi bo'lganligi uchun (1)-(3) masala yechimi mavjud va yagonadir.

3-hol: $1<\alpha<2,\ \beta=1$ bo'lsin.

4-teorema. Agar quyidagi shartlar bajarilgan bo'lsa,

1) $\nu(x)=x^{\alpha-2}\nu_1(x),\ \nu_1(x)\in C^1(\overline{J}),\ \nu_1(0)\neq 0.$

2) $\tau(x)=x^{\sigma}\tau_1(x),\ \tau_1(x)\in C^1(\overline{J})\cap C^3(J)$, $\alpha-1<\sigma<\alpha$

3) $a(x)$, $b(x)$, $p(x)$, $q(x)\in C^1(\overline{J})$, $2p(x)+b(x)\neq 0$

u holda (1)-(3) masala cheksiz ko'p yechimga ega bo'ladi.

Isboti. Bu holda (4) tenglik quyidagi ko'rinishga o'tadi

$$[2p(x)+b(x)]\nu(x)+a(x)\frac{d}{dx}D_{0x}^{\alpha-2}\nu(x)=\gamma(x), \quad 0<x<1 \quad (17)$$

Quyidagi belgilashni kiritamiz,

$$\phi(x) = D_{0x}^{\alpha-2}\nu(x), \tag{18}$$

Hosil qilingan (18) tenglikning har ikki tomoniga $D_{0x}^{2-\alpha}$ teskari operatorni qo'llaymiz. Natijada

$$\nu(x) = D_{0x}^{2-\alpha}\phi(x), \tag{19}$$

tenglikni olamiz. Bundan (18) va (19) tengliklarni (17) qo'llab

$$a(x)\frac{d}{dx}\phi(x) + [2p(x) + b(x)]\frac{d}{dx}D_{0x}^{1-\alpha}\phi(x) = \gamma(x), \tag{20}$$

tenglikni hosil qilamiz.

Ba'zi sodda almashtirishlarni bajarib, (20) tenglikni

$$a(x)\frac{d}{dx}\phi(x) + \frac{[2p(x)+b(x)](\alpha-1)}{\Gamma(\alpha-1)}x^{-1}\int_0^x (x-t)^{\alpha-2}\phi(t)dt +$$

$$+\frac{2p(x)+b(x)}{\Gamma(\alpha-1)}x^{-1}\int_0^x (x-t)^{\alpha-2}t\phi'(t)dt = \gamma(x), \tag{21}$$

ko'rinishda yozish mumkin.

$$\phi(x) = D_{0x}^{\alpha-2}\nu(x) = \frac{1}{\Gamma(2-\alpha)}\int_0^x (x-t)^{1-\alpha}\nu(t)dt = \frac{1}{\Gamma(2-\alpha)}\int_0^x (x-t)^{1-\alpha}t^{\alpha-2}\nu_1(t)dt =$$

$$= \left\langle \begin{array}{c} t = xz \\ (x-t) = (1-z)x \end{array} \right\rangle = \frac{1}{\Gamma(2-\alpha)}x^{1-\alpha+\alpha-1}\int_0^1 z^{\alpha-2}(1-z)^{1-\alpha}\nu_1(xz)dz,$$

Bundan

$$\phi(0) = \frac{\nu_1(0)}{\Gamma(1-\alpha)}B(\alpha-1, 2-\alpha) \neq 0$$

(18) shartdan hamda 4-teoremaning (1) shartidan quyidagi kelib chiqadi.

$$\phi(0) = \frac{\nu_1(0)}{\Gamma(1-\alpha)}B(\alpha-1, 2-\alpha) \neq C_0 \neq 0 \tag{22}$$

U holda, agar

$$\psi(x) = \frac{d}{dx}\phi(x) \tag{23}$$

belgilash kiritsak (22) ga asosan

$$\phi(x) = C_0 + \int_0^x \psi(t)dt, \tag{24}$$

tenglikni olamiz.

Agar (23),(24) tengliklarni (21) ga qo'ysak,

$$a(x)\psi(x) + \frac{[2p(x)+b(x)](\alpha-1)}{\Gamma(\alpha-1)x}\int_0^x (x-t)^{\alpha-2}dt\int_0^t \psi(t_1)dt_1 +$$

$$+\frac{2p(x)+b(x)}{\Gamma(\alpha-1)x}\int_0^x (x-t)^{\alpha-2}t\psi(t)dt = g(x) + \gamma(x), \tag{25}$$

$$g(x) = \frac{[2p(x)+b(x)](\alpha-1)}{\Gamma(\alpha-1)x}C_0\int_0^x (x-t)^{\alpha-2}dt \tag{26}$$

tenglik hosil bo'ladi.

Ba'zi almashtirishlardan keyin,

$$\psi(x) + \int_0^x K_3(x,t)\psi(t)dt = \gamma_3(x), \tag{27}$$

$$K_3(x,t)=\frac{\left[2p(x)+b(x)\right]x^{\alpha-2}}{\Gamma(\alpha-1)a(x)}+\frac{2p(x)+b(x)(x-t)^{\alpha-2}}{\Gamma(\alpha-1)a(x)x}t, \quad \gamma_3(x)=\frac{\gamma(x)-g(x)}{a(x)} \qquad (28)$$

Hosil bo'lgan (27) tenglikdan ikkinchi tur Volterra integral tenglamasi kelib chiqadi. U holda (4) va (27) tenglamalar ekvivalent bo'lganligi uchun (4) tenglama ham yechimga ega bo'ladi. Bundan (1)-(3) masala ham yechimga ega ekanligi va uning yechimi (6) formula bilan aniqlanganligi kelib chiqadi.

Qo'yilgan masala yechimining yagona emasligini isbotlash uchun (4) tenglamaga mos bir jinsli tenglama

$$2p(x)\nu(x)+a(x)D_{0x}^{\alpha-1}\nu(x)+b(x)D_{x1}^{\beta-1}\nu(x)=0, \qquad (29)$$

trivial bo'lmagan yechimga ega ekanligini ko'rsatish yetarli bo'ladi.

Bir qator almashtirishlardan keyin o'ng tomoni (28) tenglik bilan aniqlanadigan (27) ko'rinishdagi ikkinchi tur Volterra integral tenglamasini hosil qilamiz.

$$\psi(x)+\int_0^x K_3(x,t)\psi(t)dt=g_1(x), \qquad (30)$$

Bu yerda $g_1(x)\neq 0, \quad x\in\overline{J}$

Demak, (29) tenglamaning trivial bo'lmagan yechimi mavjud. Shuning uchun ham bir jinsli bo'lmagan (4) tenglama cheksiz ko'p yechimga ega bo'ladi. Teorema isbotlandi.

Foydalanilgan adabiyotlar.

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