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ISSIQLIK TARQALISH TENGLAMASINING FUNDAMENTAL YECHIMI VA UNING XOSSALARI

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Annotatsiya : *Ushbu maqolada tor tebranish tenglamasi uchun chegaraviy masalada kasr tartibli differensial operator qatnashgan siljishli masalani yechimining mavjudligi va yagonaligi batafsil o'rganilgan.*

Kalit so'zlar: *Kasr tartibli differensial operator, regulyar yechim, giperbolik tipdagi tenglama, Koshi masalasi, yechimning mavjudligi, Volterra integral tenglamasi, yechimning yagonaligi.*

Parabolik tipdagi tenglamalarning eng sodda vakili bo'lgan ushbu

$$u_{xx} - u_t = 0 \qquad (1)$$

tenglamani qaraylik. (1) tenglama ko'plab fizik va biologik jarayonlarni tavsiflaydi. Masalan, bir jinsli sterjenda issiqlik tarqalish jarayonini tavsiflaydi. U odatda issiqlik tarqalish tenglamasi yoki **Fure tenglamasi** deb ataladi.

Odatda ikki juft argumentga bog'liq bo'lgan ushbu

$$E(x,t,\xi,\eta) = \frac{1}{2\sqrt{\pi(t-\eta)}} \exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right], \quad t > \eta \qquad (2)$$

funksiya (1) tenglamaning fundamental yechimi deyiladi.

$E(x,t;\xi,\eta)$ funksiya quyidagi xossalarga ega:

1. x,t o'zgaruvchilar bo'yicha (1) tenglamani qanoatlantiradi.

$$\frac{\partial E}{\partial t} = \left[\frac{(x-\xi)^2}{8\sqrt{\pi}(t-\eta)^{5/2}} - \frac{1}{4\sqrt{\pi}(t-\eta)^{3/2}}\right] \exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right]$$

$$\frac{\partial E}{\partial x} = -\frac{x-\xi}{4\sqrt{\pi}(t-\eta)^{3/2}} \exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right]$$

$$\frac{\partial^2 E}{\partial x^2} = \left[\frac{(x-\xi)^2}{8\sqrt{\pi}(t-\eta)^{5/2}} - \frac{1}{4\sqrt{\pi}(t-\eta)^{3/2}}\right] \exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right]$$

Bulardan darxol $E_{xx} - E_t = 0$ kelib chiqadi

2. ξ,η o'zgaruvchilar bo'yicha (1) ga qo'shma bo'lgan $\vartheta_{\xi\xi}+\vartheta_{\eta}=0$ tenglamani qanoatlantiradi. Haqiqatan ham,

$$\frac{\partial E}{\partial \eta}=\left[\frac{1}{4\sqrt{\pi}(t-\eta)^{3/2}}-\frac{(x-\xi)^2}{8\sqrt{\pi}(t-\eta)^{5/2}}\right]\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right]$$

$$\frac{\partial E}{\partial \xi}=\frac{x-\xi}{4\sqrt{\pi}(t-\eta)^{3/2}}\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right]$$

$$\frac{\partial^2 E}{\partial \xi^2}=\left[-\frac{1}{4\sqrt{\pi}(t-\eta)^{3/2}}+\frac{(x-\xi)^2}{8\sqrt{\pi}(t-\eta)^{5/2}}\right]\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right]$$

bo'lib, bulardan darhol $E_{\xi\xi}+E_{\eta}=0$ tenglik kelib chiqadi.

3. $x\neq\xi$ bo'lganda $\lim\limits_{\eta\to t}E(x,t;\xi,\eta)=0$ tenglik o'rinli.

Bu xossani isbotlash maqsadida $E(x,t;\xi,\eta)$ funksiyani

$$E(x,t;\xi,\eta)=\left\{\frac{\dfrac{1}{2\sqrt{\pi(t-\eta)}}}{\exp\left[\dfrac{(x-\xi)^2}{4(t-\eta)}\right]}\right\}$$

ko'rinishda yozsak, $\eta\to t$ da $\frac{\infty}{\infty}$ aniqmaslikka ega bo'lamiz. Shuning uchun Lopital teoremasiga ko'ra

$$\lim_{\eta\to t}E(x,t;\xi,\eta)=\lim_{\eta\to t}\left\{\frac{\dfrac{1}{4\sqrt{\pi}(t-\eta)^{3/2}}}{\dfrac{(x-\xi)^2}{4(t-\eta)^2}\exp\left[\dfrac{(x-\xi)^2}{4(t-\eta)}\right]}\right\}=\frac{1}{\sqrt{\pi}}\lim_{\eta\to t}\left\{\frac{\sqrt{t-\eta}}{(x-\xi)^2}\exp\left[-\frac{(x-\xi)^2}{4(t-\eta)}\right]\right\}=0$$

4. $\forall\varphi(x)\in C[0;l]$ funksiya uchun quyidagi tenglik o'rinli.

$$\lim_{\eta\to t}\int_0^l E(x,t;\xi,\eta)\varphi(\xi)d\xi=\begin{cases}\frac{1}{2}\varphi(0),\ agar\quad x=0\ bo'lsa\\ \varphi(x),\quad agar\ \ x\in(0,l)\,bo'lsa\\ \frac{1}{2}\varphi(l),\ agar\ \ x=l\ \ bo'lsa\end{cases}\qquad(3)$$

(3) tenglikning chap tomonini integrallashda integralda $\frac{x-\xi}{2\sqrt{t-\eta}}=Z$ ya'ni $\xi=x-2Z\sqrt{t-\eta}$ almashtirish bajaramiz. U holda $d\xi=-2\sqrt{t-\eta}dZ$ va quyidagi

$$\xi=0\Rightarrow Z=\frac{x}{2\sqrt{t-\eta}};\qquad \xi=l\Rightarrow\ Z=\frac{x-l}{2\sqrt{t-\eta}}$$

tengliklar o'rinli bo'ladi. Bularni inobatga olsak,

$$\int_0^l E(x,t;\xi,\eta)\varphi(\xi)d\xi=\frac{1}{\sqrt{\pi}}\int_{\frac{x-l}{2\sqrt{t-\eta}}}^{\frac{x}{2\sqrt{t-\eta}}} e^{-Z^2}\varphi\left(x-2Z\sqrt{t-\eta}\right)dZ \tag{4}$$

tenglikka ega bo'lamiz.

Matematik analizdan ma'lum bo'lgan

$$\int_{-\infty}^{+\infty} e^{-Z^2}dZ=2\int_0^{+\infty} e^{-Z^2}dZ=\int_{-\infty}^{0} e^{-Z^2}dZ=\sqrt{\pi}$$

tengliklarni e'tiborga olib, (4) tengliklarda η ni t ga intiltirib limitga o'tamiz:

$$\lim_{\eta\to t}\int_0^l E(x,t;\xi,\eta)\varphi(\xi)d\xi=\frac{1}{\sqrt{\pi}}\lim_{\eta\to t}\int_{\frac{x-l}{2\sqrt{t-\eta}}}^{\frac{x}{2\sqrt{t-\eta}}} e^{-Z^2}\varphi\left(x-2Z\sqrt{t-\eta}\right)dZ$$

$$\begin{cases}\frac{1}{\sqrt{\pi}}\varphi(0)\cdot\int_{-\infty}^{0} e^{-Z^2}dZ=\frac{1}{2}\varphi(0),\ agar\quad x=0\ bo'lsa\\ \frac{1}{\sqrt{\pi}}\varphi(x)\cdot\int_{-\infty}^{0} e^{-Z^2}dZ=\varphi(x),\quad agar\ x\in(0,l)\,bo'lsa\\ \frac{1}{\sqrt{\pi}}\varphi(l)\cdot\int_{-\infty}^{0} e^{-Z^2}dZ=\frac{1}{2}\varphi(l),\ agar\ x=l\ bo'lsa\end{cases}$$

Foydalanilgan adabiyotlar.

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