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PARABOLIK–GIPERBOLIK TIPDAGI TENGLAMALAR UCHUN XARAKTERISTIKADAN SILJIGAN CHIZIQLARNI O`Z ICHIGA OLGAN QUYI YARIM SOHADA CHEGARAVIY MASALA

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Annotatsiya : *Ushbu maqolada parabolik–giperbolik tipdagi tenglama uchun xarakteristikadan siljigan chiziqlarni o`z ichiga olgan quyi yarim sohalarda chegaraviy masala uchun qo'yilgan nolokal shartli chegaraviy masala yechimining mavjudligi va yagonaligi isbotlangan*

Kalit so'zlar: *Parabolik–giperbolik tipdagi tenglama, xarakteristik uchburchak, regulyar yechim, integral energiya usuli, trivial yechim, Grin funksiyasi, Volterraning ikkinchi tur integral tenglamasi, Dalamber formulasi.*

1. Masalaning qo`yilishi

Quyidagi tenglamani qaraymiz:

$$0 = Lu \equiv \begin{cases} u_{xx} - u_y, & (x,y) \in \Omega_0, \\ u_{xx} - u_{yy}, & (x,y) \in \Omega_j \left(j = \overline{1,3} \right), \end{cases} \tag{1}$$

bu yerda Ω_0 soha deb $x > 0,\ y > 0$ bo`lganda $y = 0,\ \ x = 1,\ \ y = 1,\ \ x = 0$ to`g'ri chiziqlarda mos ravishda joylashgan $AB,\ BB_0,\ B_0A_0,\ A_0A,$ kesmalar bilan chegaralangan to`rtburchak sohani, Ω_1 soha $x < 0,\ y > 0$ da ΔAA_0D xarakteristik uchburchakning ichida joylashgan $AK : x = \gamma_1(y)$ silliq egri chiziq va (1) tenglamanig $BP:\ y - x = 1$ xarakteristikasi bilan chegaralangan soha, Ω_2 soha $x > 0,\ y > 0$ da ΔBB_0E xarakteristik uchburchakning ichida joylashgan $AC : x = -\gamma_2(y)$ silliq egri chiziq va (1) tenglamanig $B_0M:\ x + y = 2$ xarakteristikasi bilan chegaralangan soha, Ω_3 soha $x < 0,\ y < 0$ da (1) tenglamanig $AC:\ y + x = 0$ va $BC:\ y - x = -1$ xarakteristikasi bilan chegaralangan $\Delta\, AA_0D$ xarakteristik uchburchak soha,

Quyidagi belgilashlarni kiritamiz: $J_1 = \{(x,y):\ 0 < x < 1,\ \ y = 0\},$

$J_2 = \{(x,y):\ x = 0, 0 < y < 1\},\ J_3 = \{(x,y):\ x = 1,\ \ 0 < y < 1\},$

$\Omega=\Omega_0\cup\Omega_1\cup\Omega_2\cup\Omega_3\cup J_1\cup J_2\cup J_3$, $C\left(-\frac{1}{2},\frac{1}{2}\right)$, $D\left(-\frac{1}{2},\frac{1}{2}\right)$, $E\left(\frac{3}{2},\frac{1}{2}\right)$,

$\theta\left(\frac{x}{2};-\frac{x}{2}\right)$, $\left[\theta^*\left(\frac{\lambda(x)+x}{2};\frac{\lambda(x)-x}{2}\right)\right]$

bu yerda $\theta_2(x)\left[\theta_2^*(x)\right]$ (1) tenglamaning $(x,0)\in J_3$ nuqtadan chiquvchi xarakteristikalari bilan $AC[AN]$ xarakteristikalarining kesishish nuqtasining koordinatalari.

A-Shart. $y=-\gamma_1(x)$ va $\gamma_j(y)(j=2,3)$ - berilgan funksiyalar bo`lib quyidagi shatlarni bajarsin:

1) $\gamma_1(x)$ va $\gamma_j(y)(j=2,3)$ funksiyalar mos ravishda ΔACB va $\Delta AA_0D,\ \Delta BEB_0$ xarakteristik uchburchaklar ichidaqa to`liq joylashgan bo`lsin;

2) $\gamma_1(x)\in C^2(0,1),\ \gamma_j(y)\in C^2(0,1)(j=2,3)$ tegishli bo`lsin;

3) $t\pm\gamma_j(t)\left(j=\overline{1,3}\right)$ - monoton o'suvchi;

4) $\gamma_1(0)=0,\ \gamma_2(0)=-1,\ l_1+\gamma_1(l_1)=1,\ l_2-\gamma_2(l_2)=2, l_3+\gamma_3(l_3)=1,\ l_j=const$ $l_j\in\left(\frac{1}{2};1\right)$.

Ta'rif. (1) tenglamaning Ω sohadagi regulyar yechimi deb,

$$W_1=\left\{u:u(x,y)\in C^1\left(\overline{\Omega}\right)\cap C_{x,y}^{2,1}(\Omega_0)\cap C^2(\Omega_i), i=\overline{1,3}\right\}$$

sinfga tegishli bo'lgan hamda $\Omega_i\left(i=\overline{0,3}\right)$ sohada (1) tenglamani qanoatlantiruvchi $u(x,y)$ yechimga aytiladi.

I – Masala. Quyidagi shartlarni qanoatlantiruvchi (1) tenglamaning regulyar yechimi topilsin:

$$\left[u_x-u_y\right]\theta(x)+\mu(x)\left[u_x-u_y\right]\theta^*(x)=\varphi(x); \quad (2)$$

$$u|_{A_0D}=g_1(y);\quad u|_{B_0E}=g_2(y) \quad (3)$$

$$\left.\left[u_x+u_y\right]\right|_{AD}=p(y); \quad (4)$$

$$\left.\left[u_x-u_y\right]\right|_{BE}=q(y); \quad (5)$$

$$u(A)=u(B)=0; \quad (6)$$

Bunda $\mu(x),\ \varphi(x),\ g(y),\ p(y)$ va $q(y)$ - berilgan yetarlicha silliq funksiyalar.

Teorema. Agar $\mu(x)\neq-1;\ \mu(x),\varphi(x)\in C^1[0,1]\left(i=\overline{1,3}\right)$, $g(y),\ p(y),\ q(y)\in C^2(0;1)$ va **A** shartlar bajarilsa, I - masalaning yagona regulyar yechimi mavjud bo'ladi.

Isbot. Ω_1 sohalarda Koshi masalasining yechimining ko'rinishi quyidagicha bo'ladi.

$$u(x,y)=\frac{1}{2}\left[\tau_1(x+y)+\tau_1(x-y)+\int_{x-y}^{x+y}\nu_1(t)dt\right],\ 0<x<1$$

Bundan $\theta\left(\frac{x}{2};-\frac{x}{2}\right)$ va $\theta^*\left(\frac{\lambda(x)+x}{2};\frac{\lambda(x)-x}{2}\right)$ nuqtalarda quyidagi funksional munosabatlarni olamiz:

$$(1+\mu(x))\nu_1(x)=[1+\mu(x)]\tau_1'(x)-\varphi(x),\ 0<x<1 \qquad (7)$$

(1) dan Ω_0 sohada $y\to+0$ limitga o'tib quyidagi tenglamani olamiz.

$$(\tau_1(x))''=\nu_1(x) \qquad (8)$$

(7) ni (8) ga qo'yib $\tau_1(x)$ ga nisbatan ikkinchi tartibli oddiy differensial tenglamani hosil qilishimiz mumkin.

$$(\tau_1(x))''-\tau_1'(x)=-\frac{\varphi(x)}{1+\mu(x)} \qquad (9)$$

(9) tenglamani $\tau_1(0)=0$ va $\tau_1(1)=0$ shartlar ostida yechimning umumiy ko'rinishining shakli tasvirlanadi.

$$\tau_1(x)=\int_0^x\frac{\varphi(t)\left[1-e^{x-t}\right]}{1+\mu(t)}dt+\frac{e^x-1}{e-1}\int_0^1\frac{\varphi(t)\left[e^{1-t}-1\right]}{1+\mu(t)}dt \qquad (10)$$

(8) dan foydalanib, $\nu_1(x)$ ni topamiz

$$\nu_1(x)=-\int_0^x\frac{\varphi(t)e^{x-t}}{1+\mu(t)}dt+\frac{e^x}{e-1}\int_0^1\frac{\varphi(t)\left(e^{1-t}-1\right)}{1+\mu(t)}dt-\frac{\varphi(x)}{1+\mu(x)} \qquad (11)$$

Integral tenglamalar metodi bilan masalaning yechimini mavjudligini isbotlaymiz. Buning uchun biz (8)- (9) funksional munosabatlardan va Ω_0 sohada (1) tenglama uchun qo`yilgan birinchi chegaraviy masalaning yechimidan

$$u(x,y)=\int_0^1\tau_1(t)G(x,y;t,0)dt+\int_0^y\tau_2(t)G_t(x,y;0,z)dz-$$
$$-\int_0^y\tau_3(z)G_t(0,y;1,z)dz, \qquad (12)$$

ko'rinishda bo'ladi.

Bu yerda $G(x,y;t,z)=\frac{1}{2\sqrt{\pi(y-z)}}\sum_{n=-\infty}^{\infty}\left[e^{-\frac{(x-t+2n)}{4(y-z)}}-e^{-\frac{(x+t+2n)}{4(y-z)}}\right]$ - issiqlik o'tkazuvchanlik tenglamasi uchun birinchi chegaraviy masalaning Grin funksiyasi.

$\tau_k(y)$, $\nu_k(y)$ $(k=2,3)$ funksiyalar orasidagi munosabat olish uchun bir marta x bo'yicha differensiallab:

$$u_x(x,y)=\int_0^1 \tau_1(t)G_x(x,y;t,0)dt+\int_0^y \tau_2(z)G_{tx}(x,y;0,z)dz-$$
$$-\int_0^y \tau_3(z)G_{tx}(x,y;1,z)dz,$$

(13)

ni hosil qilamiz. Quyidagi $N(x,y;t,z)=\frac{1}{2\sqrt{\pi(y-z)}}\sum_{n=-\infty}^{\infty}\left[e^{-\frac{(x-t+2n)^2}{4(y-z)}}+e^{-\frac{(x+t+2n)^2}{4(y-z)}}\right]$ belgilash kiritsak,

$G_{tx}(x,y;t,z)=N_z(x,y;t,z),\ G_x(x,y;t,z)=-N_t(x,y;t,z)$ munosabatlarga ega bo'lamiz. $u_x(0,y)=\nu_2(y)$ belgilashga ko'ra (13) dan quyidagi munosabatlarni olamiz:

$$\nu_2(y)=\int_0^1 \tau_3'(z)N(x,y;1,z)dt-\int_0^y \tau_1'(t)N(x,y;t,0)dt+\int_0^y \tau_2'(z)N(0,y;0,z)dz$$
$$\nu_3(y)=\int_0^1 \tau_3'(z)N(1,y;1,z)dz-\int_0^y \tau_1'(z)N(1,y;t,0)dz+\int_0^y \tau_2'(z)N(1,y;0,z)dz \qquad (14)$$

Ma'lumki, $u_{xx}-u_{yy}=0$ tenglamaning umumiy yechimi

$$u(x,y)=f_1(x+y)+f_2(x-y) \qquad (15)$$

ko'rinishda bo'ladi, bunda $f_1(.),\ f_2(.)$ - ikkinchi tartibli uzluksiz differensiallanuv – chi noma'lum funksiya.

(4) shartidan va (15) dan $f_1'(y-\gamma_2(y))=p(y),\ 0\le y\le l$ ga ega bo'lamiz, tenglamadan $y-\gamma_2(y)=t$ ni yechini $y=\delta_1(t)$ ko'rinishda izlab

$f_1'(t)=\frac{1}{2}p(\delta_1(t)),\ 0\le y\le l$, bundan

$$f_1(y)=f_1(0)+\frac{1}{2}\int_0^y p(\delta(t))dt,\ 0\le y\le l.$$

(3.8) shartidan va (15) dan $f_2'(y-\gamma_2(y))=q(y),\ 0\le y\le l$ ga ega bo'lamiz, tenglamadan $y-\gamma_2(y)=t$ ni yechini $y=\delta_1(t)$ ko'rinishda izlab

$f_2'(t)=\frac{1}{2}q(\delta_1(t)),\ 0\le y\le l$, bundan

$$f_2(y)=f_2(0)+\frac{1}{2}\int_0^y q(\delta(t))dt,\ 0\le y\le l.$$

Endi $l\le y\le 1$ da $u|_{A_0D}=g_1(y)$ va $u|_{B_0E}=g_2(y)$ shartni hisobga olsak,

$$\begin{cases} f_1(y)=g_1\left(\frac{y-1}{2}\right)+f_2(1),\ l\le y\le 1 \\ f_2(y)=g_2\left(\frac{2-y}{2}\right)+f_1(2),\ l\le y\le 1 \end{cases}$$

(15) ga $f_1(y)$ va $f_2(y)$ ning qiymatni qo'yamiz va quyidagiga ega bo'lamiz.

$$u(x,y)=\begin{cases} f_2(x-y)+\frac{1}{2}\int_0^y p(\delta(t))dt+f_1(0), 0\le y\le l,\\ f_2(x-y)+g_1\left(\frac{y-1}{2}\right)+f_2(1),\ l\le y\le 1.\\ f_1(x+y)+\frac{1}{2}\int_0^y q(\delta(t))dt+f_2(0), 0\le y\le l,\\ f_1(x+y)+g_2\left(\frac{2-y}{2}\right)-f_1(2),\ l\le y\le 1.\end{cases} \tag{16}$$

$u_y(0,y)=\tau_i'(y),\ i=2,3$ ligidan, (16) tenglikni y bo'yicha bir marta differensiallab $x\to 0$ desak

$$\tau_2'(y)=\begin{cases} -f_2'(-y)+\frac{1}{2}p(\delta(y)), & 0\le y\le l,\\ -f_2'(-y)+\frac{1}{2}g_1'\left(\frac{y-1}{2}\right), & l\le y\le 1.\end{cases} \tag{17}$$

$x\to 1$ da esa

$$\tau_3'(y)=\begin{cases} f_1'(1+y)+\frac{1}{2}q(\delta(1+y)), & 0\le y\le l,\\ f_1'(1+y)-\frac{1}{2}g_2'\left(\frac{1-y}{2}\right), & l\le y\le 1.\end{cases} \tag{18}$$

(16) ni (17)va (18) ga qo'yib $f_i'(y)\ i=2,3$ funksiya uchun $\tau_i(y)\ i=2,3$ va $\nu_i(y)\ i=2,3$ funksiyalar o'rtasida quyidagi funksional munosabatni olamiz:

$$\begin{cases} \tau_2'(y)=\nu_2(y)+p(\delta(y)), & 0\le y\le l,\ x<0\\ \tau_2'(y)=\nu_2(y)+g_1'\left(\frac{y-1}{2}\right), & l\le y\le 1.\ x<0\end{cases} \tag{19}$$

$$\begin{cases} \tau_3'(y)=\nu_3(y)+q(\delta(1-y)), 0\le y\le l,\ x>1\\ \tau_3'(y)=\nu_3(y)+g_2'\left(\frac{1-y}{2}\right),\ l\le y\le 1, \quad x>1\end{cases} \tag{20}$$

(19)va (20) dan $\nu_2(y)$ va $\nu_3(y)$ ni (14) ga qo'ysak

$$\tau_2'(y)-\int_0^1 \tau_3'(z)N(x,y;1,z)dt+\int_0^y \tau_1'(t)N(x,y;t,0)dt-\int_0^y \tau_2'(z)N(0,y;0,z)dz=F_2(y)$$

$$\tau_3'(y)-\int_0^1 \tau_3'(z)N(1,y;1,z)dz+\int_0^y \tau_1'(z)N(1,y;t,0)dz-\int_0^y \tau_2'(z)N(1,y;0,z)dz=F_3(y) \tag{21}$$

Bu yerda

$$F_2(y)=\frac{1}{2}p(\delta(1-y))-\frac{1}{2}g_1^{'}\left(\frac{1-y}{2}\right)$$

$$F_3(y)=\frac{1}{2}q(\delta(1-y))-\frac{1}{2}g_2^{'}\left(\frac{1-y}{2}\right)$$

(21) sistemani

$$\begin{cases}\tau_2^{'}(y)-\int\limits_0^y\tau_2^{'}(z)N(0,y;0,z)dz=F_2^*(y),\\ \tau_3^{'}(y)+\int\limits_0^y\tau_3^{'}(z)N(1,y;1,z)dz=F_3^*(y).\end{cases} \quad (22)$$

ko'rinishda olamiz, bunda

$$\begin{cases}F_2^*(y)=F_2(y)+\int\limits_0^y\tau_3^{'}(z)N(x,y;1,z)dz-\int\limits_0^1\tau_1^{'}(t)N(x,y,t,0)dt,\\ F_3^*(y)=F_3(y)+\int\limits_0^y\tau_2^{'}(z)N(1,y;0,z)dz-\int\limits_0^y\tau_1^{'}(z)N(1,y,t,0)dt\end{cases} \quad (23)$$

$$|N(0,y,0,z)|\le\frac{2}{\sqrt{\pi|y-y_1|}}\sum_{n=1}^{\infty}\left|e^{-\frac{n^2}{|y-y_1|}}\right|\le const$$

(14) sistema

$\left|F_2^*(y)\right|\le const$ bo'lgani uchun (23) sistemadan 1 – tenglamani ketma – ket yaqinlashish usuli bilan yechib

$$\tau_2(y)=F_2^*(y)+\int\limits_0^y F_2^*(t)K(t,y)dt \quad (24)$$

ni olamiz.

$F_2^*(y)$ ni (24) ga qo'yib

$$\tau_2(y)=F_2(y)+\int\limits_0^y\tau_3^{'}(z)N(x,y;1,z)dz-\int\limits_0^1\tau_1^{'}(t)N(x,y,t,0)dt+$$

$$+\int\limits_0^y\left(F_2(y)+\int\limits_0^t\tau_3^{'}(z)N(x,y;1,z)dz-\int\limits_0^y\int\limits_0^1\tau_1^{'}(p)N(x,y,p,0)dp\right)K(t,y)dt \quad (25)$$

Va nihoyat (25) ni (21) ning 2 – tenglamasiga qo'yamiz va $\tau_2(y)$ ga nisbatan Volterra ikkinchi tur integral tenglamasini hosil qilamiz:

$$\tau_3^{'}(y)-\int\limits_0^y\tau_3^{'}(z)N(1,y;1,z)dz+\int\limits_0^y\tau_1^{'}(z)N(1,y;t,0)dz-\int\limits_0^z N(1,y;0,z)dz-\int\limits_0^y\tau_3^{'}(z)N(x,y;1,z)+$$

$$+\int\limits_0^y\tau_3^{'}(z)N(x,y;1,z)dz-\int\limits_0^y\int\limits_0^1\tau_1^{'}(p)N(x,y,p,0)dpK(t,y)dp+F_3(y)=F_2(y) \quad (26)$$

bu yerda

$$F_3(y)=\int_0^y F_2(z)N(1,y,0,z)dz+\int_0^y F_2(s)K(s,y)ds+\int_0^y N(1,y,0,z)\int_0^y F_2(p)K(p,y)dp$$

(27) (26) tenglamani yechib $\tau_2(y)$, bundan va (25) dan $\tau_1(y)$ va (26), (27)dan $\nu_1(y)$ va $\nu_2(y)$ ni topamiz. $\tau_i(y),\nu_i(y)\,(i=\overline{1,3})$ lar ma'lum funksiyalar. Endi Ω_0 sohada I masalaning yechimini tiklashimiz mumkin, $\overline{\Omega_i}\,(i=\overline{1,3})$ sohalarda esa Koshi masalasining yechimi bo'lgan Dalamber formulasi orqali yechim topiladi. Demak, I-masala bir qiymatli yechildi.

Teorema isbotlanadi.

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