

SONNING BUTUN VA KASR QISMI TUSHUNCHASI VA UNING TADBIQLARI

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Annotatsiya : Matematikaning ko'pgina tadbiqlarida haqiqiy sonning butun qismi va kasr qismi tushunchasi bilan ish ko'rishga to'g'ri keladi. Ushbu ishda sonning butun va kasr qism tushunchasi va uning tatdiqlari o'rganilgan.

Kalit so'zlar: butun qism, kasr qism, tub son, tub ko'paytuvchi.

Ta'rif – 1. x haqiqiy sonning butun qismi deb, x dan oshmaydigan eng kata butun songa aytiladi va u $[x]$ simvol orqali belgilanadi.

Masalan: $[4,3] = 4$, $[\pi] = 3$, $[-5,2] = -6$, $[-\pi] = -4$

$f(x) = [x]$ funksiyaning xossalari:

1°. $D[x] = R' = (-\infty, \infty)$ – barcha haqiqiy sonlar to'plami;

2°. $E[x] = Z$ – barcha butun sonlar to'plami;

3°. $\forall x_1, x_2$ haqiqiy sonlar uchun $[x_1 + x_2] \geq [x_1] + [x_2]$;

4°. $\forall x \in R, \forall n \in Z$ uchun $[x + n] = [x] + n$;

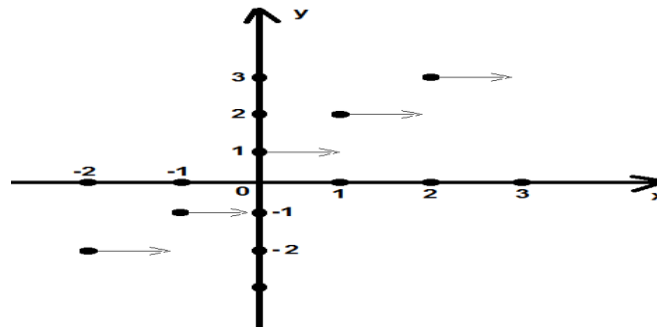
5°. Agar $[x] = [y]$ bo'lsa, $|x - y| < 1$.

6°. $\forall x$ haqiqiy son uchun $[x] \leq x < [x] + 1$;

7°. $\forall n \in N, \forall x \in R$ uchun $\left[\frac{[x]}{n} \right] = \left[\frac{x}{n} \right]$.

Bu funksiya x ning barcha butun qiymatlarida uzilishga ega bo'ladi va grafigi 1-chizmadan iborat.





1-chizma.

Ta'rif – 2. x ning haqiqiy sonning kasr qismi deb, x bilan uning $[x]$ butun qismi orasidagi ayirmaga aytiladi va u $\{x\}$ simvol bilan belgilanadi. Demak, $\{x\} = x - [x]$.

Masalan: $\{4,3\} = 4,3 - [4,3] = 4,3 - 4 = 0,3$ $\{-5,2\} = -5,2 - [-5,2] = -5,2 + 6 = 0,8$

Ravshanki, har qanday x sonini uning butun va kasr qismlari yig'indisi ko'rinishida tasvirlash mumkin:

$$x = [x] + \{x\}.$$

$g(x) = \{x\}$ funksiyaning xossalari:

1^o. $D\{x\} = R = (-\infty, \infty)$;

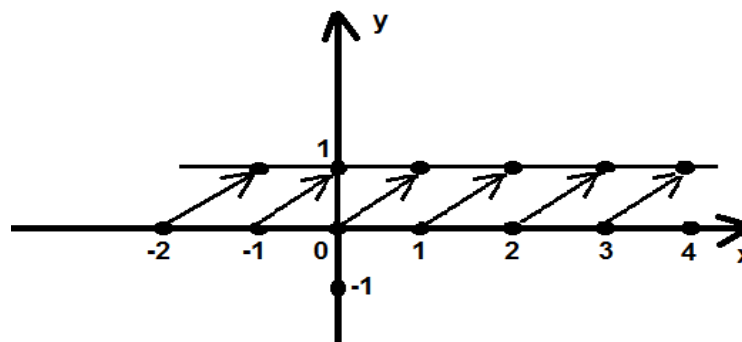
2^o. $E\{x\} = (0; 1]$

3^o. $\forall x$ haqiqiy son uchun $\{[x]\} = 0$ va $[\{x\}] = 0$, shuningdek $[[x]] = [x]$, $\{ \{x\} \} = \{x\}$ tengliklar o'rinli.

4^o. $g(x) = \{x\}$ funksiya asosiy davri $T = 1$ ga teng davriy funksiyadir, ya'ni $\{x\} = \{x + 1\}$

tenglik istalgan x haqiqiy son uchun bajariladi, $\forall n \in \mathbb{Z}$ uchun $\{x + n\} = \{x\} + n$.

5^o. $\{x\} = \{y\}$ bo'lsa, $x - y$ ayirma butun son bo'ladi. Bu funksiya x ning barcha butun qiymatlarida uzilishga ega bo'ladi va grafigi 2-chizmadan iborat.



2-chizma.

Misollar qarayliz.

1-misol. Ushbu $[x + [x]] = 113$ tenglamani yeching.

Yechilishi: $[x + n] = [x] + n$ munosabatga asosan $[x + [x]] = [x] + [x] = 2 \cdot [x]$ juft son bo'lib, u 113 ga teng bo'lishi mumkin emas. demak, tenglamaning yechi yo'q.

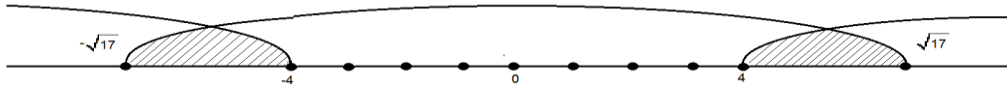


2-misol. $\lceil x^2 \rceil = 16$ tenglamani yech.

Yechilishi: Ravshanki, $16 \leq x^2 < 17$ bo'ladi. Bu tenglamalarni yechamiz.

$$x^2 \geq 16 \quad (x-4)(x+4) \geq 0 \Rightarrow (-\infty; -4] \cup [4; \infty)$$

$$x^2 < 17, \quad (x - \sqrt{17})(x + \sqrt{17}) < 0 \Rightarrow -\sqrt{17} < x < \sqrt{17}$$



Javob: $(-\sqrt{17}; -4] \cup [4; \sqrt{17})$

3-misol. $x^2 2\{x\} - 5 = 0$ tenglamani yech.

Yechilishi: Agar $0 \leq \{x\} < 1$ ni etiborga olsak, berilgan. $2\{x\} = x^2 - 5, \quad 5 \leq x^2 < 7,$

Yoki $\sqrt{5} \leq |x| < \sqrt{7}$ bundan $[x] = 2$ yoki $[x] = -3$ bo'lib, $\{x\} = x - [x]$ tenglikka asosan berilgan tenglamani ushbu ikkita tenglamani yechishga keltiramiz:

$$1^{\circ}. \quad x^2 - 2(x - 2) - 5 = 0, \quad 2^{\circ}. \quad x^2 - 2(x + 3) - 5 = 0.$$

Birinchi tenglamani yechib: $x_{1,2} = 1 \pm \sqrt{2}$ ni hosil qilamiz $[x] = 2$ shartni $x = 1 + \sqrt{2}$ ildiz qanoatlantiradi. Ikkichi tenglamani yechib $[x] = -3$ shartni qanoatlantiradi $x = 1 - 2\sqrt{3}$ ildizni hosil qilamiz.

Javob: $x = 1 + \sqrt{2}$ va $x = 1 - 2\sqrt{3}$.

4-misol. $\left\lceil \frac{x}{3} \right\rceil^2 - 3 \left\lceil \frac{x}{3} \right\rceil - 4 = 0$ tenglamani qanoatlantiruvchi nechta turli butun son bor?

Yechilishi: Agar $\left\lceil \frac{x}{3} \right\rceil = t$ desak, u holda $t^2 - 3t - 4 = 0$ tenglama hosil bo'ladi, uni yechib $t_1 = -1, \quad t_2 = 4$ ildizlarni topamiz, natijada

$\left\lceil \frac{x}{3} \right\rceil = -1$ va $\left\lceil \frac{x}{3} \right\rceil = 4$ tenglamalar hosil bo'ladi. Tarifga asosan bu tenglamalarni yechsak:

$$-1 \leq \frac{x}{3} < 0, \quad -3 \leq x < 0, \quad x: -3; -2; -1.$$

$$4 \leq \frac{x}{3} < 5, \quad 12 \leq x < 15, \quad x: 12, 13, 14.$$

Javob: -3; -2; -1; 12; 13; 14 yani, berilgan tenglamani qanoatlantiruvchi butun sonlar 6 ta ekan.

5-misol. $\left\lceil \left\lceil \frac{x+2}{3} \right\rceil \right\rceil = 2$ bo'lsa, quyidagilarning qaysi biri to'g'ri?

A) $8 \leq x < 11$ B) $7 \leq x < 10$ C) $6 \leq x < 9$ D) $5 \leq x < 8$ E) $4 \leq x < 7$.



Yechilishi: $\left[\left[x \right] \right] = \left[x \right]$ tenglikka asosan, $\left[\frac{x+2}{3} \right] = 2$ tenglikka ega bo'lamiz, haqiqiy sonning butun qismining xossasiga ko'ra $2 \leq \frac{x+2}{3} < 3$ tenglikka ega bo'lamiz, uni yechib $4 \leq x < 7$ yechimini hosil qilamiz.

Javob: E) $4 \leq x < 7$.

6-misol. 30 dan kata bo'lmagan barcha nuqtalar sonlarning ko'paytmasi $k (k \in \mathbb{N})$ ning qanday eng katta qiymatida 2^k ga qoldiqsiz bo'linadi?

Yechilishi: Sonning butun qismi qo'llaniladigan quyudagi lejandr teoremasini qaraymiz:

Teorema: p tub soni $n!$ sonining tub ko'paytuvchilariga yoyilmasida

$\left[\frac{n}{p} \right] + \left[\frac{n}{p^2} \right] + \left[\frac{n}{p^3} \right] + \dots$ daraja ko'rsatgichi bilan qatnashadi, yetarli kata R lar ($k \rightarrow \infty$) uchun

$\left[\frac{n}{p^k} \right] = 0$ Endi $1 \cdot 2 \cdot 3 \cdot \dots \cdot 30$ ko'paytmada 2 tub ko'paytuvchi necha marta qatnashishini nisoblaymiz: Lejandr teoremasiga binoan

$$\left[\frac{30}{2} \right] + \left[\frac{30}{2^2} \right] + \left[\frac{30}{2^3} \right] + \left[\frac{30}{2^4} \right] = 15 + 7 + 3 + 1 = 26.$$

Javob: $k=26$.

7-misol. $[\lg x] \cdot \{\lg x\} = \lg x$ tenglamani yeching.

Yechilishi: Deylik, $[\lg x] = n$, $n \in \mathbb{Z}$ va $\{\lg x\} = d$, $0 \leq d < 1$ bo'lsin. U holda $\lg x = [\lg x] + \{\lg x\}$ formulaga asosan $\lg x = n + d$ bo'ladi. Berilgan tenglamadan esa

$n \cdot d = n + d$ tenglikka ega bo'lamiz, bu yerdan $n = \frac{d}{d-1}$ yoki $d = \frac{n}{n-1}$ ni topamiz.

$n \leq 0, n \in \mathbb{Z}$ ekanligi kelib chiqadi. Shunday qilib, $\lg x = \frac{n^2}{n-1}$, bundan esa $x = 10^{\frac{n^2}{n-1}}$ hosil bo'ladi.

Javob: $x = 10^{\frac{n^2}{n-1}}$, $n \leq 0, n \in \mathbb{Z}$.

Xulosa o'rnida shuni aytish mumkinki, haqiqiy sonning butun va kasr qismi tushunchalarining tadbirlar kengayib bormoqda, sababdan ham bu tushunchalarni va ular bilan bog'liq barcha bog'lanishlarni o'rganish dolzarb masaladir.

Foydalanilgan adabiyotlar.

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