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MARKAZIY KUCH MAYDONIDA TEBRANISH BILAN HARAKATLANAYOTGAN JISM UCHUN HARAKAT TENGLAMASI

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Annotation *Tortuvchi kuch ta'sirida aylanma harakatga qilayotgan jism o'z aylanish o'qi atrofida vertikal ravishda teng ravishda tebranishi mumkin. Yuqoriga va pastga vertikal tebranish, albatta, jisimning aylanish koordinatasiga qo'shimcha ravishda boshqa umumiy koordinatalarga ega bo'lishiga olib keladi.*
Biz ushbu ishda tortuvchi markaziy kuchning tebranish ta'siridan vujudga keladigan vertikal tebranish harakatining ta'sirini analitik va sifat jihatidan ko'rib chiqdik.
Keywords. *Elliptik tekislik, vertikal tebranishlar, kritik tezlik*

Markaziy kuchlar saqlanuvchidir kuchlardir. Uning kattaligi va yo'nalishi masofaga bog'liq va markazga yoki unga qarama-qarshi yo'nalgan bo'ladi. Markaziy maydon tushunchasini ifodalashda qutub koordinatalar sistemasidan foydalanish hisoblashlarni osonlashtiradi, shu sababli markaziy kuchni $F = f(r)\hat{r}$ deb tanlab olamiz. Bunda $f(r) < 0$ bo'lganida tortuvchi, $f(r) > 0$ bo'lganida, itariluvchi kuchlarni ifodalaydi.

Misol uchun tortuvchi markaziy kuchlarga Quyosh hosil qilgan gravitatsion markaziy kuchlarini aytish mumkin. Bundan tashqari elektronlarni atomga bog'laydigan yadro kuchlari shubhasiz markaziy xususiyatga ega.

Markaziy kuch maydonida muvozanat atrofida tebranayotgan jismlarning harakat tenglamasini yozishda quyidagi vektordan foydalanamiz:

$$\vec{r} = r\hat{r} = r\hat{r}(\theta, \beta, \mu, \alpha, \phi), \tag{1}$$

yuqoridagi vektordan vaqt bo'yicha to'liq hosila olamiz:

$$\upsilon = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\left(\frac{d\hat{r}}{d\theta}\frac{d\theta}{dt} + \frac{d\hat{r}}{d\beta}\frac{d\beta}{dt} + \frac{d\hat{r}}{d\mu}\frac{d\mu}{dt} + \frac{d\hat{r}}{d\alpha}\frac{d\alpha}{dt} + \frac{d\hat{r}}{d\phi}\frac{d\phi}{dt}\right) \tag{2}$$

$$\upsilon = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\beta}\hat{\beta} + r\dot{\mu}\hat{\mu} + r\dot{\phi}\hat{\phi} \tag{3}$$

(3) ifodadan vaqt bo'yicha hosila olib tezlanishni topamiz, ya'ni:

$$a=\ddot{r}\hat{r}+\dot{r}\left(\frac{d\hat{r}}{d\theta}\frac{d\theta}{dt}+\frac{d\hat{r}}{d\beta}\frac{d\beta}{dt}+\frac{d\hat{r}}{d\mu}\frac{d\mu}{dt}+\frac{d\hat{r}}{d\alpha}\frac{d\alpha}{dt}+\frac{d\hat{r}}{d\phi}\frac{d\phi}{dt}\right)$$

$$+\dot{r}\dot{\theta}\hat{\theta}+r\ddot{\theta}\hat{\theta}+r\dot{\theta}^2\left(\frac{d\hat{\theta}}{d\theta}\right)+\dot{r}\dot{\beta}\hat{\beta}+r\ddot{\beta}\hat{\beta}+r\dot{\beta}^2\left(\frac{d\hat{\beta}}{d\beta}\right)+\dot{r}\dot{\mu}\hat{\mu}+r\ddot{\mu}\hat{\mu}$$

$$+r\dot{\mu}^2\left(\frac{d\hat{\mu}}{d\mu}\right)+\dot{r}\dot{\alpha}\hat{\alpha}+r\ddot{\alpha}\hat{\alpha}+r\dot{\alpha}^2\left(\frac{d\hat{\alpha}}{d\alpha}\right)+\dot{r}\dot{\phi}\hat{\phi}+r\ddot{\phi}\hat{\phi}+r\dot{\phi}^2\left(\frac{d\hat{\phi}}{d\phi}\right) \quad (4)$$

$$a=\left(\ddot{r}-r\dot{\theta}\right)\hat{r}+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\hat{\theta}+\left(r\ddot{\beta}+2\dot{r}\dot{\beta}-r\dot{\beta}^2\tan\beta\right)\hat{\beta}$$

$$+\left(r\ddot{\mu}+2\dot{r}\dot{\mu}-r\dot{\mu}^2\tan\mu-r\dot{\mu}^2 ctag\mu\right)\hat{\mu}+\left(r\ddot{\alpha}+2\dot{r}\dot{\alpha}-r\dot{\alpha}^2\tan\alpha\right)\hat{\alpha}+$$

$$\left(r\ddot{\phi}+2\dot{r}\dot{\phi}-r\dot{\phi}^2\tan\phi-2r\dot{\phi}^2 ctag\phi\right)\hat{\phi} \quad , (4')$$

β - yuqori radial orbital tebranish burchagi va α -pastki radial orbital tebranish burchagi. (4') -tenglama vertikal tebranish effekti qo‘shilganda markaziy kuch ta’sirida harakatlanyotgan jismning tezlanish ifodalovchi yangi tenglamasidir.

Navbatda kuchni topamiz, bunda kuchni koordinatani funksiyasi sifatida qaraymiz:

$$f(r)=m\left\{\left(\ddot{r}-r\ddot{\theta}\right)-\left(r\dot{\beta}^2\tan\beta+r\dot{\alpha}^2\tan\alpha\right)\right\} \quad (5)$$

$$m\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)=0,\ m\left(r\ddot{\mu}+2\dot{r}\dot{\mu}\right)=0,\ m\left(r\ddot{\beta}+2\dot{r}\dot{\beta}\right)=0,$$

$$m\left(r\ddot{\alpha}+2\dot{r}\dot{\alpha}\right)=0,\ m\left(r\ddot{\phi}+2\dot{r}\dot{\phi}\right)=0 \quad (6)$$

(6) tenglamalar harakat miqdori momentini ifodalab $\theta,\beta,\mu,\alpha\ va\ \phi$ koordinatalar oshib borishi bilan oshadi.

Orbital burchak tezlanish yo‘nalishida kuch ta’sir etmaaydi, chunki kuch markaziy xarakterga ega ekanligidan, faqat radus bo‘yicha yo‘nalgan bo‘ladi. Tangensial tezlanish tebranish fazasini kuchga aylantirib natijani nolga tenglab topamiz:

$$m\left(r\ddot{\mu}+2\dot{r}\dot{\mu}-r\dot{\mu}^2\tan\mu-2r\dot{\mu}^2 ctag\mu\right)=0$$

$$m\left(r\ddot{\phi}+2\dot{r}\dot{\phi}-r\dot{\phi}^2\tan\phi-2r\dot{\phi}^2 ctag\phi\right)=0 \quad (7)$$

(7) tanglamaning ikkal ifodasi bir biriga o‘xshash ekanligidan birinichi tenglaamani yechish bilan kifoyalanamiz:

$$\frac{m}{r}\left(r^2\ddot{\mu}+2r\dot{r}\dot{\mu}-r^2\dot{\mu}^2\tan\mu-r^2\dot{\mu}^2 ctag\mu\right)=0 \quad (8)$$

(8) tenglamani shaklini quyidagicha o‘zgartiramiz:

$$\frac{d}{dt}\left(r^2\dot{\mu}\right)-\frac{d}{dt}\int\left(r^2\dot{\mu}^2\tan\mu+2r^2\dot{\mu}^2 ctag\mu\right)$$

bundan quyidaqini olamiz:

$$\frac{d}{dt}\left(r^2\dot{\mu}\right)-\frac{d}{dt}\int\left(r^2\dot{\mu}^2\tan\mu+2r^2\dot{\mu}^2 ctag\,\mu\right)\Rightarrow$$

$$\frac{d}{dt}\left\{\left(r^2\dot{\mu}\right)-\int\left(r^2\dot{\mu}^2\tan\mu+2r^2\dot{\mu}^2 ctag\,\mu\right)\right\}=0$$

$$\left(r^2\dot{\mu}\right)-\int\left(r^2\dot{\mu}^2\tan\mu+2r^2\dot{\mu}^2 ctag\,\mu\right)=E_I \qquad (9)$$

(7) ni ikkinch tenglamasi uchun E_I ga o‘xshash ifodani olamiz:

$$\left(r^2\dot{\phi}\right)-\int\left(r^2\dot{\phi}^2\tan\phi+2r^2\dot{\phi}^2 ctag\,\phi\right)=E_{II}\,. \qquad (10)$$

Umumiy tebranish energiyasini topamiz:

$$E_{teb}=r^2\left(\dot{\mu}+\dot{\phi}\right)-\int r^2\left(\dot{\mu}^2\tan\mu+\dot{\phi}^2\tan\phi\right)-2\int r^2\left(\dot{\mu}^2 ctag\,\mu+\dot{\phi}^2 ctag\,\phi\right), \qquad (11)$$

tebranish energiyasi radius vektorining funksiyasi bo‘lib, vertikal tebranish burchaklari ortishi bilan u manfiy bo‘ladi.

(7) tenglamalarni yechaylik, bund biz yuqoridagi tenglamani kvadrat tenglamaga o‘xshash usulda yechamiz, bunda $m\neq 0$ bo‘lsin:

$$\left(r\ddot{\mu}+2\dot{r}\dot{\mu}-r\dot{\mu}^2\tan\mu-2r\dot{\mu}^2 ctag\,\mu\right)=0 \qquad (12)$$

$$\dot{\mu}=\frac{2\dot{r}\pm\sqrt{\left(4\dot{r}^2+4\dot{r}^2\ddot{\mu}\left(\tan\mu+2ctag\,\mu\right)\right)}}{2r\left(\tan\mu+2ctag\,\mu\right)}\,. \qquad (13)$$

Ildiz ostidagi 4$\dot{r}$ ni chiqarib qayta yozamiz:

$$\dot{\mu}=\frac{2\dot{r}\pm 2\dot{r}\sqrt{\left(1+\frac{r^2}{\dot{r}^2}\ddot{\mu}\left(\tan\mu+2ctag\,\mu\right)\right)}}{2r\left(\tan\mu+2ctag\,\mu\right)}, \qquad (14)$$

(14) diskriminantni nolga tenglab quyidagini topamiz:

$$D=1+\frac{r^2}{\dot{r}^2}\ddot{\mu}\left(\tan\mu+2ctag\,\mu\right)=0 \qquad (15)$$

$$\ddot{\mu}\left(\tan\mu+2ctag\,\mu\right)=-\frac{\dot{r}^2}{r^2}\Rightarrow\dot{r}^2=-r^2\ddot{\mu}\left(\tan\mu+2ctag\,\mu\right)$$

yuqoridagi tenglikdan $\dot{r}$ ni topamiz:

$$\dot{r}=-r\sqrt{\ddot{\mu}\left(\tan\mu+2ctag\,\mu\right)}, \qquad (16)$$

shunday qilib, radial tezlik radius vektoriga to‘g‘ri proportsional va vertikal tebranish burchaklarining kvadrat ildiziga ham to‘g‘ri proportsional proportsionaldir. Vertikal aylanishning tebranish burchaklari ortishi bilan radial tezlik kamayadi deb xulosa qilish mumkin.

Jismning kinetik energiyasini (3) tenglamadan foydalanib topamiz:

$$T=\frac{1}{2}m\upsilon^2=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\dot{\beta}^2+r^2\dot{\mu}^2+r^2\dot{\phi}^2\right). \qquad (17)$$

Harakat tenglamasini topish uchun olingan ifodalarni Lagrang tenglamasiga qo‘yamiz:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=0 \qquad (18)$$

(18) tenglamadagi q va $\dot{q}$ lar mos ravishda umumlashgan koordinata va umumlashgan tezlik bo‘lib, $q=(r,\theta,\beta,\mu,\alpha,\phi)$, $\dot{q}=(\dot{r},\dot{\theta},\dot{\beta},\dot{\mu},\dot{\alpha},\dot{\phi})$ hamda $L=T-U(r)$ ekanligini esga olib, topamiz:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}-\frac{\partial U(r)}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}+\frac{\partial U(r)}{\partial q}=\Rightarrow\left|\frac{\partial U(r)}{\partial \dot{q}}=0,\ \frac{\partial U(r)}{\partial q}=\frac{dU}{dq}\right|\Rightarrow$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}+\frac{dU}{dq}=0 \qquad (19)$$

mos hosilalarini o‘rniga qo‘ysak:

$$\frac{d}{dt}(m\dot{r})-mr\left(\dot{\theta}^2+\dot{\beta}^2+\dot{\mu}^2+\dot{\alpha}^2+\dot{\phi}^2\right)-f(r)=0 \qquad (20)$$

(19) ifodan ikkinchi va uchunchi hadlaridan umumlashgan koordinata bo‘yicha olingan hosilalarini nol desak, sodda tenglamani olamiz:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)=0, \qquad (21)$$

bu yerdan ba’zi algebraik algebraik almashtirishlarni bajarsak quyidagilar kelib chiqadi:

$$\frac{d}{dt}\left(mr^2\dot{\theta}\right)=0;\ \left(mr^2\dot{\theta}\right)=l;\ \dot{\theta}=\frac{l}{mr^2}$$
$$\frac{d}{dt}\left(mr^2\dot{\beta}\right)=0;\ \left(mr^2\dot{\beta}\right)=l;\ \dot{\beta}=\frac{l}{mr^2}$$
$$\frac{d}{dt}\left(mr^2\dot{\mu}\right)=0;\ \left(mr^2\dot{\mu}\right)=l;\ \dot{\mu}=\frac{l}{mr^2}$$
$$\frac{d}{dt}\left(mr^2\dot{\alpha}\right)=0;\ \left(mr^2\dot{\alpha}\right)=l;\ \dot{\alpha}=\frac{l}{mr^2}$$
$$\frac{d}{dt}\left(mr^2\dot{\phi}\right)=0;\ \left(mr^2\dot{\phi}\right)=l;\ \dot{\phi}=\frac{l}{mr^2}. \qquad (22)$$

Yuqoridagi tenglamalar m massali jisimning Lagranj harakat tenglamalar to‘plami ekan. (22) ifodalarni (20) ni mos o‘rniga qo‘yib quyidagi ifodani olamiz:

$$\frac{d}{dt}(m\dot{r})-5mr\left(\frac{l}{mr^2}\right)^2-f(r)\Rightarrow\frac{d}{dt}(m\dot{r})-\frac{5l^2}{mr^3}-f(r)=0 \qquad (23)$$

(23) tenglamani ikki tamonini $\dot{r}$ ga ko‘paytirib quyidagini hosil qilamiz:

$$\frac{d}{dt}(m\dot{r})\dot{r}-\frac{5l^2}{mr^3}\dot{r}-f(r)\dot{r}=0 \qquad (24)$$

hosil bo‘lgan ifodani dt ga ko‘paytirib olamiz:

$$d\left(m\dot{r}^2\right)-\frac{5l^2}{mr^3}dr-f(r)dr=0 \quad (25)$$

(25) ni integrallab energiyani topamiz:

$$\left(\frac{1}{2}m\dot{r}^2\right)+\frac{5l^2}{2mr^2}-\int f(r)dr=E_r \quad (26)$$

(26) tenglama jismning tangensial tebranish hamda markaziy nuqta atrofida aylanib yurishda ega bo‘lgan radial energiyasini beradi. Yuqoridagi mulohazalardan E_t jismning umumiy energiyasi, aylanma harakat radial energiya E_r va tebranish energiyasi E_{teb} yig‘indisi bo‘lar ekan, ya’ni:

$$E_t=r^2\left(\dot{\mu}+\dot{\phi}\right)-\int r^2\left(\dot{\mu}^2\tan\mu+\dot{\phi}^2\tan\phi\right)-2\int r^2\left(\dot{\mu}^2 ctag\mu+\dot{\phi}^2 ctag\phi\right)$$
$$+\left(\frac{1}{2}m\dot{r}^2\right)+\frac{5l^2}{2mr^2}-\int f(r)dr \quad (27)$$

(27) dan koo‘rinadaiki, tebranish energiyasi E_t uchta mustaqil umumlashtirilgan koordinatadan va ikkita asosiy qismdan iborat bo‘lar ekan.

Xulosa qilib biz ushbu maqolada jismning ma’lum markaziy tortishish hamda qo‘shimcha ravishda tangensial ta’sir etuvchi tebranish kuchiga bog‘liq bo‘lgan harakati masalasini qarab chiqdik. Bu holat uchun jismning tezligini, energiyasini hisoblash formulalarini keltirib chiqardik.

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