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Two Points for a Second-Order Inhomogenously Destructive Ordinary Differential Equation Solution of the 4th Boundary Limitation Problem Using the Method of Green's Functions

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Annotation: In the article boundary value problem was investigated for second order ordinary differential equation with singular coefficients. The uniqueness and existence of the solution of the considered problem was proved.

Keywords: boundary value problem, second order differential equation with singular coefficients, uniqueness of the solution, existence of the solution.

Statement of issue $[0; p]$ continuous in segment, following

$$\acute{o}'' + \frac{2\gamma}{x} y' + \lambda y = f(x), \qquad x \in (0, p) \qquad (1)$$

the differential equation and

$$\lim_{x \to 0} x^{2\gamma} y'(x) = 0, \quad y'(p) = 0 \qquad (2)$$

find *y(x)* a function satisfying homogeneous boundary conditions.

Theorem. If $0 < \gamma < \frac{1}{2}$, $\lambda > 0$, $J_{1/2+\gamma}\left(\sqrt{\lambda} p\right)$ if, then {(1),(2)} the solution to the problem will be available and unique.

Proof. (1) is homogeneous according to Equation

$$\left(x^{2\gamma} y'\right)' + \lambda x^{2\gamma} y = 0 \qquad (1')$$

general solution of an equation of the form

$$\acute{o}(x) = C_1 x^{\frac{1}{2}-\gamma} J_{1/2-\gamma}\left(\sqrt{\lambda} x\right) + C_2 x^{\frac{1}{2}-\gamma} J_{\gamma-1/2}\left(\sqrt{\lambda} x\right). \qquad (3)$$

found in the view. So, we look for the Green's function of the problem {(1),(2)} in the following form:

$$G(x,s) = \begin{cases} \alpha_1 (x)^{\frac{1}{2}-\gamma} J_{1/2-\gamma}\left(\sqrt{\lambda} x\right) + \alpha_2 (x)^{\frac{1}{2}-\gamma} J_{\gamma-1/2}\left(\sqrt{\lambda} x\right), & 0 \le x \le s, \\ \beta_1 (x)^{\frac{1}{2}-\gamma} J_{1/2-\gamma}\left(\sqrt{\lambda} x\right) + \beta_2 (x)^{\frac{1}{2}-\gamma} J_{\gamma-1/2}\left(\sqrt{\lambda} x\right), & s \le x \le p. \end{cases} \qquad (4)$$

Subjecting (4) to conditions (2), we obtain the following:

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$$G(x,s)=\begin{cases}\alpha_2(x)^{\frac{1}{2}-\gamma}J_{\gamma-1/2}\left(\sqrt{\lambda}x\right), & 0\le x\le s,\\ \beta_2(x)^{\frac{1}{2}-\gamma}\left[J_{\gamma-1/2}\left(\sqrt{\lambda}x\right)+\dfrac{J_{\gamma+1/2}\left(\sqrt{\lambda}p\right)}{J_{-1/2-\gamma}\left(\sqrt{\lambda}p\right)}J_{1/2-\gamma}\left(\sqrt{\lambda}x\right)\right], & s\le x\le p.\end{cases}$$

$x=s$ at the point $G(x,s)$ fom the continuity of the function and $G'_x(x,s)$ get up $1/s^{2\gamma}$ using the fact that it has a jump,

$$\begin{cases}\alpha_2(s)^{\frac{1}{2}-\gamma}J_{\gamma-1/2}\left(\sqrt{\lambda}s\right)-\beta_2(s)^{\frac{1}{2}-\gamma}\left[J_{\gamma-1/2}\left(\sqrt{\lambda}s\right)+\dfrac{J_{\gamma+1/2}\left(\sqrt{\lambda}p\right)}{J_{-1/2-\gamma}\left(\sqrt{\lambda}p\right)}J_{1/2-\gamma}\left(\sqrt{\lambda}s\right)\right]=0, & 0\le t\le s,\\ -\alpha_2\sqrt{\lambda}(s)^{\frac{1}{2}-\gamma}J_{\gamma+1/2}\left(\sqrt{\lambda}s\right)-\beta_2\sqrt{\lambda}(s)^{\frac{1}{2}-\gamma}\left[\dfrac{J_{\gamma+1/2}\left(\sqrt{\lambda}p\right)}{J_{-1/2-\gamma}\left(\sqrt{\lambda}p\right)}J_{-1/2-\gamma}\left(\sqrt{\lambda}s\right)-J_{\gamma+1/2}\left(\sqrt{\lambda}s\right)\right]=\dfrac{1}{s^{2\gamma}}, & s\le t\le p.\end{cases}$$

we create a system of equations. From this system α_2 and β_2 we determine the Green's function of the problem {(1),(2)} as follows:

$$G(x,s)=\begin{cases}\dfrac{\pi(xs)^{\frac{1}{2}-\gamma}}{2\cos\gamma\pi}\dfrac{J_{\gamma-1/2}\left(\sqrt{\lambda}x\right)}{J_{1/2+\gamma}\left(\sqrt{\lambda}p\right)}\left[J_{\gamma-1/2}\left(\sqrt{\lambda}s\right)J_{-\gamma-1/2}\left(\sqrt{\lambda}p\right)+J_{1/2-\gamma}\left(\sqrt{\lambda}s\right)J_{1/2+\gamma}\left(\sqrt{\lambda}p\right)\right], & 0\le x\le s,\\ \dfrac{\pi(sx)^{\frac{1}{2}-\gamma}}{2\cos\gamma\pi}\dfrac{J_{\gamma-1/2}\left(\sqrt{\lambda}s\right)}{J_{\gamma+1/2}\left(\sqrt{\lambda}p\right)}\left[J_{\gamma-1/2}\left(\sqrt{\lambda}x\right)J_{-\gamma-1/2}\left(\sqrt{\lambda}p\right)+J_{1/2-\gamma}\left(\sqrt{\lambda}x\right)J_{1/2+\gamma}\left(\sqrt{\lambda}p\right)\right], & s\le x\le p.\end{cases}\quad(5)$$

(5) the general solution of the problem {(1),(2)}

$$y(x)=\int_0^p G(x,s)s^{2\gamma}f(s)ds,$$

is determined by appearance. The theorem is proved.

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