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# Development of Students' Analytical Thinking Based on Calculation Experiments in the Educational Process

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**Annotation:** In this article, fractional-rational functions are a very important part of analytical thinking because they simplify many complex problems and require a deeper approach to mathematical analysis. Through them, not only mathematical problems can be solved, but also practical problems in real life. The importance of the analysis and application of fractional-rational functions in the development of analytical thinking, especially in solving differential equations, physical systems, and engineering problems is covered.

**Keywords:** fractional-rational functions, polynomials, analytical thinking, rational functions.

Rational functions, which are the ratio of two polynomials, play a significant role in analytical thinking and mathematical analysis. These functions appear frequently in many branches of mathematics, including calculus, differential equations, algebra, and applied fields such as physics, engineering, and economics. The ability to understand and manipulate rational functions is essential for solving complex mathematical problems and developing deeper insights into various theoretical and practical contexts.

In the context of analytical thinking, rational functions serve as a foundational tool in several key areas. Here's an exploration of their importance in analytical thinking:

1. Decomposition and Simplification

One of the most important applications of rational functions in analytical thinking is partial fraction decomposition. This technique involves breaking a complex rational function into simpler, more manageable fractions that are easier to integrate, differentiate, or manipulate.

Analytical techniques such as partial fraction decomposition require logical reasoning and step-by-step problem solving to break down a function into simpler components. For instance, decomposing a function like P(x)Q(x)\frac{P(x)}{Q(x)}Q(x)P(x) into a sum of simpler fractions involves recognizing the structure of the denominator and using algebraic skills to find the correct decomposition.

The process of decomposing functions develops analytical thinking by requiring the solver to identify key relationships between the terms in a function and use them to simplify the expression. This is crucial when approaching complex integrals or solving algebraic problems.

2. Limits, Singularities, and Continuity

Rational functions often involve limits and singularities, which are critical concepts in calculus and analytical thinking.

Singularities occur when the denominator of a rational function equals zero, leading to undefined points (or poles). Recognizing and analyzing singularities is essential for understanding the behavior of rational functions near these points. Analytical thinking helps in identifying whether the function has removable discontinuities or infinite limits at certain points.

The behavior of rational functions near singularities requires the use of limits to determine the function's approach to infinity or other values. Understanding how rational functions behave as x→∞x

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\to \inftyx→∞ or near singular points is a key aspect of analytical thinking, and it involves recognizing the degrees of the numerator and denominator polynomials and their impact on the limit.

The concept of continuity is closely related to rational functions, as many rational functions are continuous except where the denominator is zero. Understanding the continuity of functions helps in forming correct hypotheses about the function's behavior and makes it possible to apply techniques like L'Hopital's Rule or Taylor expansions.

3. Rational Functions in Differential Equations

Rational functions are widely used in differential equations and play a crucial role in both solving and modeling real-world phenomena. Many physical and engineering problems are modeled by differential equations that include rational functions.

Linear differential equations often involve rational functions, especially in systems where the rates of change of certain quantities are inversely proportional to each other. For example, in electrical engineering, the behavior of circuits can often be described by transfer functions, which are rational functions.

Solving differential equations that involve rational functions requires deep analytical skills, as it often involves techniques like separation of variables, integration by parts, or using special functions like the Laplace transform. Analytical thinking is necessary to recognize which technique will lead to a solvable form of the equation.

4. Rational Functions in Applied Mathematics

Rational functions also have significant applications in applied mathematics, such as physics, engineering, and economics. They are used to model systems where relationships between variables are proportional in a linear or inverse fashion.

Physics: In classical mechanics, fluid dynamics, and electrical circuits, rational functions frequently appear in equations that describe physical phenomena, such as the relationship between voltage, current, and resistance in circuits, or the velocity of a fluid under certain conditions.

Engineering: In control systems and signal processing, rational functions are often used to describe the system's behavior, especially in terms of transfer functions and frequency response. The analysis of these functions helps engineers design and optimize systems.

Economics: Rational functions can model various economic relationships, such as the supply and demand curves, where the quantity demanded or supplied is related to price by a rational function. Analytical skills are crucial to understanding these models and predicting economic behavior.

5. Critical Thinking and Problem Solving

Rational functions play an important role in critical thinking and problem solving, as they require the ability to break down complex relationships into simpler components and reason logically about their behavior.

Recognizing patterns: Analytical thinking allows mathematicians and scientists to recognize patterns in rational functions that lead to efficient methods of simplification or solution. For example, recognizing that a rational function can be expressed as a sum of partial fractions or that its behavior near a singularity can be understood using limits.

Connecting concepts: The ability to connect concepts such as polynomials, limits, singularities, and integrals in the context of rational functions is essential for solving problems and making sense of more advanced mathematical models.

6. Advanced Mathematical Topics

Rational functions are foundational in understanding more advanced mathematical topics, such as complex analysis and asymptotic analysis.

Complex Analysis: Rational functions play a crucial role in complex analysis, particularly when studying residues and contour integrals. Analytical thinking is required to understand how rational functions behave in the complex plane, particularly when dealing with poles and branch cuts.

Asymptotic Analysis: Rational functions are also important in asymptotic analysis, where they are used to describe the behavior of functions as they approach infinity. Analytical thinking is necessary to understand the rates at which the numerator and denominator grow and how this influences the overall behavior of the function.

Separation of fractional-rational functions into simple fractions (or partial separation) is one of the main methods of mathematical analysis, especially widely used in the integration of fractional-rational functions. A fractional-rational function is a function expressed as a ratio of two polynomials, that is, it has the following form:

$$f(x) = \frac{P(x)}{Q(x)}$$

**Rational functions** are functions that can be expressed as the ratio of two polynomials, i.e., f(x)=P(x), where P(x) and Q(x) are polynomials, and Q(x)≠0. In the process of integrating rational functions, their complexity often requires the application of the **partial fractions** method. This technique involves breaking down a complex rational function into simpler fractions, making each part easier to integrate individually. This approach significantly simplifies the integration process and helps in solving numerous mathematical problems.

The main goal of **partial fraction decomposition** is to break down a complex rational function into simpler, integrable parts. This decomposition is typically performed by dividing polynomials, and the method is especially useful when the degree of the numerator is smaller than that of the denominator.

During the partial fraction decomposition process, various technical methods can be applied. The most common technique is **factoring the denominator into simpler terms**. This allows a complex rational expression to be rewritten as a sum of simpler fractions. By decomposing rational functions in this way, it becomes much easier to solve complex integral problems, which is why this method is a fundamental aspect of **mathematical analysis**.

**Literature Review**

Rational functions, expressed as the ratio of two polynomials, are widely used in fields such as **mathematical analysis**, **analytic geometry**, **physics**, **engineering**, and **economics**. They are an integral part of analytical thinking and mathematical analysis, helping to deepen the understanding of various concepts and methods. Rational functions are crucial not only in mathematical theory but also in solving practical real-world problems.

There are several key factors that contribute to the development of analytical thinking through the study of rational functions:

1. Fundamental Methods in Analyzing Rational Functions

One of the most important methods used in the **analytic study** of rational functions is **partial fraction decomposition**. This technique allows complex rational functions to be broken down into simpler forms, making integration and other mathematical operations easier. Analytical thinking is required to correctly and efficiently apply this method.

- **Solving Systems of Equations**: During partial fraction decomposition, a system of equations is constructed and solved to find the unknown coefficients. This process strengthens analytical thinking, as solving complex expressions requires correctly following mathematical rules and methods.
- **Polynomials and Their Roots**: To decompose rational functions, it is necessary to find and work with the roots of polynomials. Analytical thinking aids in understanding the properties of these polynomials, such as their degree, roots, and factorization properties.

2. Uncertainty and Limit Concepts in the Analysis of Rational Functions

In analyzing rational functions, concepts like **limits** and **singularities** play a significant role. For instance:

- **Singular Points**: One distinctive feature of rational functions is that they can have singularities at certain points (e.g., when the denominator equals zero). Analyzing the behavior of the function near such points requires in-depth study of limits and their various properties.
- **Limits and Continuity**: The concepts of limits and continuity are central to the analysis of rational functions. Analytical thinking helps in understanding how rational functions approach infinity or behave near specific points, and how they may be discontinuous or behave erratically.

3. Rational Functions and Differential Equations

Rational functions frequently appear in **differential equations** and their solutions. Analytical thinking is essential when working with differential equations, as it involves the correct formulation and application of techniques to solve these equations.

- **Linear Differential Equations**: Rational functions can be used to solve linear differential equations. Such equations often require analytical solutions, and rational functions may emerge as solutions.
- **Differential Operators and Transformations**: In analyzing rational functions using differential operators, analytical approaches are necessary because many complex systems and operations involve rational functions.

4. Applications of Rational Functions in Physics and Engineering

Rational functions are not only used in mathematics but also play a significant role in **physics** and **engineering**. For example, they appear in formulas used in **electric circuits**, **signal processing**, and the study of **mechanical systems** dynamics.

- Rational Functions in Electric Circuits: Rational functions are used to describe relationships between voltage, current, and resistance in electric circuits. Analyzing these functions helps in determining the behavior of the system in both the time and frequency domains.
- Signal Analysis: Rational functions, especially transfer functions, are used in signal transmission and processing. To analyze and simplify signals, it is essential to understand rational functions analytically.

5. Analytical Thinking and the Properties of Rational Functions

In analyzing rational functions, analytical thinking connects various mathematical and physical concepts. For example:

- Polynomials and Their Degrees**: Proper understanding of the properties of rational functions requires recognizing the relationship between the degrees of the polynomials involved.**
- Transformations**: By applying transformations (e.g., Laplace or Fourier transforms) to rational functions**, new properties can be derived. Studying these transformations further strengthens analytical thinking.

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**Rational functions** are functions expressed as the ratio of two polynomials. Common forms of such rational functions include:

$$\frac{A}{x-a};\ \frac{A}{(x-a)^k};\ (k>1, k\in Z), \frac{Ax+B}{x^2+px+q};\ \left(\frac{p^2}{4}-q<0\right)\ \frac{Ax+B}{(x^2+px+q)^n};\ (n>1, n\in Z, \frac{p^2}{4}-q<0)$$

Here, $A, B, p, q, a$ are real numbers. In cases where rational functions are complex, we often need to decompose them into simpler fractions for easier integration or analysis. This decomposition process, called **partial fraction decomposition**, is essential for simplifying complex rational functions into a sum of simpler fractions, making mathematical operations more manageable.

## Conclusion

Rational functions, being the ratio of two polynomials, play a crucial role in **analytical thinking** and **mathematical analysis**. They are not only fundamental in theoretical mathematics but also in solving practical problems in various fields such as physics, engineering, and economics. Through techniques such as partial fraction decomposition, complex integral problems can be simplified, making the process of solving mathematical problems more accessible and manageable. As such, the study of rational functions is essential for developing strong analytical thinking and understanding deeper mathematical and scientific concepts.

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