Impact Factor: 9.9 ISSN-L: 2544-980X

To'Rtinchi Tartibdagi Differensial Tenglamalar Uchun Chegaraviy Masala Yechimi To'G'Risida

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Annotatsiya: Mazkur maqolada chegaralangan sohada to'rtinchi tartibli tenglama uchun chegaraviy masala qaralgan bo'lib, masalani yechimining yagonaligi integral energiya usulida isbotlangan. Masalani yechimi Grin funksiyalari orqali aniq topilgan.

Kalit so`zlar: Yuqori tartibli tenglama, Grin formulasi yechimning yagonaligi, mavjudligi.

Murakkab va murakkab-aralash tipdagi tenglamalar uchun ustozlarimiz M.S.Salohiddinov [2], T.D.Jo'rayev [1] va ularning shogirdlari tomonidan chegaraviy masalalar qo'yilib ularni o'rganish nazariyalari yaratilgan. Ushbu tenglama uchun D masala [3] da tahlil etilgan. Bu maqolada to'rtinchi tartibli tenglama uchun birlik aylana bilan chegaralangan sohada chegaraviy masala o'rganilgan.

Masalani qo'yilishi.

Ushbu to'rtinchi tartibli tenglamani

$$\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=0, \qquad (1.1)$$

birlik aylana bilan chegaralangan $D\subset \mathbf{R}^2$ sohaда ko'rib chiqamiz.

H masala. $u(x,y)\in C^4(D)\cap C^2(\overline{D})$ funksiya topilsinki (1.1) tenglamani va quyidagi chegaraviy shartlarni qanoatlantirsin

$$u(x,y)=f_1(x,y),\quad \frac{\partial^2 u(x,y)}{\partial n^2}=f_2(x,y),\quad (x,y)\in\Gamma, \qquad (1.2)$$

bu yerda $f_1(x,y)$, $f_2(x,y)$ –berilgan funksiyalar, n–Γ ga tashqi normal,

$\Gamma: x^2+y^2=1$ D sohani chegarasi. Γ_1 bilan Γ ning $N_1(-1,0)$ nuqtadan $N_2(1,0)$ nuqtagacha musbat yo'nalishdagi qismini belgilaymiz, qolgan qismini Γ_2 bilan belgilaymiz.

I. **H masala yechimining yagonaligi**.

Teorema 1.1. Agar H masalani yechimi mavjud bo'lsa u yagona bo'ladi.

Isboti. 1.1 teoremani isbotlash uchun (1.1) tenglamani quyidagi shartlar bilan ko'rib chiqamiz

$$u|_{\Gamma}=0,\qquad \left.\frac{\partial^2 u}{\partial^2 n}\right|_{\Gamma}=0 \quad . \qquad (1.3)$$

Ushbu $D_\varepsilon=\{(x,y)\in D;\ dist[\Gamma,(x,y)]>\varepsilon\}$ sohani qaraymiz.

[1] dotsent, FarDU, Farg`ona, O`zbekiston
[2] FarDU, Farg`ona, O`zbekiston

Faraz qilamiz $u(x,y)$ (1.1) yechimi bo'lsin. U holda (1.1) tenglamani $u(x,y)$ funksiyaga ko'paytirib D_ε sohada integrallaymiz,

$$\iint_{D_\varepsilon} u\frac{\partial^2}{\partial x^2}(u_{xx}+u_{yy})dxdy=0\,. \qquad (1.4)$$

D va D_ε sohalarda quyidagi tenglik o'rinli bo'ladi

$$u\frac{\partial^2}{\partial x^2}\Delta u=\left[(u\Delta u_x)_x-(u_x u_{xx})_x+u_{xx}^2-(u_x u_{xy})_y+u_{xy}^2\right], \quad (1.5)$$

$$\text{bu yeda } \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} - \text{Laplas operatori.}$$

Grin formulasini qo'llab, (1.4) tenglikdan (1.3) shartlarini hisobga olsak va (1.5) dan foydalanib, $\varepsilon\to 0$ da quyidagini hosil qilamiz

$$\iint_D\left[W_x^2+W_y^2\right]dxdy=0. \qquad (1.6)$$

Bu yerda

$$W(x,y)=u_x\,. \qquad (1.7)$$

Haqiqatdan ham,, Γ da quyidagi tengliklar o'rinli:

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial n^2}\cdot\left(\frac{dx}{dn}\right)^2+\frac{\partial^2 u}{\partial s^2}\cdot\left(\frac{dx}{ds}\right)^2+\frac{\partial^2 u}{\partial s\partial n}\frac{dx}{dn}\frac{dx}{ds},$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial n^2}\cdot\left(\frac{dy}{dn}\right)^2+\frac{\partial^2 u}{\partial s^2}\cdot\left(\frac{dy}{ds}\right)^2+\frac{\partial^2 u}{\partial s\partial n}\frac{dy}{dn}\frac{dy}{ds}.$$

$$\frac{\partial^2 u}{\partial y\,\partial x}=\frac{\partial^2 u}{\partial n^2}\cdot\frac{dy}{dn}\frac{dx}{dn}+\frac{\partial^2 u}{\partial s\partial n}\cdot\left(\frac{dy}{ds}\frac{dx}{dn}+\frac{dy}{dn}\frac{dx}{ds}\right)+\frac{\partial^2 u}{\partial s^2}\frac{dx}{ds}\frac{dy}{ds}.$$

$U\big|_\Gamma=0$ ekanligidan $\left.\dfrac{\partial^2 u}{\partial s^2}\right|_\Gamma=0$ kelib chiqdi. Shunga asosan va (1.3) shartga ko'ra Γ da

$$u_{xx}=0, u_{yy}=0, u_{xy}=0 \qquad (1.8)$$

tengliklarni ega bo'lamiz. Shuning uchun (1.6) dan

$$\iint_D\left(W_x^2+W_y^2\right)dxdy=0 \qquad (1.9)$$

ifodani topamiz. Bundan $W(x,y)=const$ ekanligi kelib chiqadi. $u\in C^2(\overline{D})$ bo'lganligi uchun $\overline{D}$ da $W(x,y)=0$ bo'ladi. Buni va (1.7), (1.8) larni hisobga olsak, quyidagi ifodaga ega bo'lamiz.

$$u_x=0 \qquad (1.10)$$

(1.10) tenglamani ushbu shartni

$$u(x,y)=0, \quad (x,y)\in\Gamma_1, \quad (1.11)$$

qanoatlantiruvchi yechimi D sohada aynan nolga teng. Haqiqatdan ham (1.10) dan $u=\varphi(y)$ ni yozish mumkin. Bu yerda $\varphi(y)$-ixtiyoriy funksiya. $u|_{\Gamma_1}=0$ shartga bo'ysundirilsa, u holda $u\equiv0$ ekanligi kelib chiqdi.

II. H masalaning yechimini mavjudligi.

Ushbu belgilashni kiritamiz. $f_i(s)=f_i(x(s),y(s)),\ i=1,2.$

Teorema 1.2. Agar $f_1''(s),\ f_2'(s)$ funksiya Γ chiziqda uzluksiz va $f_1''(N_i)=0,\ i=1,2.$, bo'lsa u holda masalani yechimi mavjud.

Isbot. Ushbu belgilashni kiritamiz.

$$\frac{\partial^2 u}{\partial x^2}=\upsilon(x,y). \qquad (1.12)$$

U holda (1.1) tenglama quyidagi ko'rinishga ega

$$\Delta\upsilon=0, \qquad (x,y)\in D. \qquad (1.13)$$

(1.13) tenglamaga

$$\upsilon(x,y)=\omega(x,y), \quad (x,y)\in\Gamma, \qquad (1.14)$$

chegaraviy shartni qo'yamiz, bunda funksiya $\omega(x,y)=f_2x_n^2+f_1''x_s+f_{1n}'x_nx_s$.

Ma'lumki [4] (1.13), (1.14) masalani yechimi

$$\upsilon(x,y)=\int_\Gamma \frac{\partial G(x,y,\zeta(s),\eta(s))}{\partial n}\omega(s)ds \qquad (1.15)$$

ko'rinishda ifodalanadi. Bu yerda

$$G(x,y,\zeta(s),\eta(s))=\frac{1}{2\pi}\ln\frac{1}{r}+g(x,y,\zeta,\eta)$$ - D da Grin funksiyasi,

$$r^2=(x-\zeta)^2+(y-\eta)^2,\ g(x,y,\zeta,\eta)$$ - regulyar qismi.

(1.12) tenglamadan, (1.7) belgilashga ko'ra va (1.2) shartlarni hisobga olsak, quyidagilarga ega bo'lamiz.

$$W_x=\upsilon(x,y) \qquad (1.16)$$

$$W|_\Gamma=\begin{cases} f_{11}(x,y),\ agar\ (x,y)\in\Gamma_1, \\ f_{12}(x,y),\ agar\ (x,y)\in\Gamma_2, \end{cases} \qquad (1.17)$$

bunda

$$f_{1i}(x,y)=x_nf_3(x,y)+x_sf_1'(x,y),$$

$$i=1,2,\quad x_n=\cos(x,n)=y'(s),\quad y_n=\cos(y,n)=-x'(s).$$

$f_3(x,y)$-noma'lum funksiya

(1.16) tenglamani (1.17) shartni qanoatlantiruvchi yechimi

$$W(x,y)=\frac{1}{\pi}\int_{\Gamma}\eta'(s)\ln\left((x-\zeta(s))^2+\right.$$

$$\left.+(y-\eta(s))^2\right)\omega(s)ds+\int_{\Gamma}g_1(x,y,\zeta(s),\eta(s))\omega(s)ds+$$

$$+f_{12}\left(x,\mu_2\left(y\right)\right),\ (1.18)$$

ko'rinishda bo'ladi, bu yerda $\mu_2(y)=\sqrt{1-y^2}$. Bu yerda

$$g_1\left(x,y,\zeta\left(s\right),\eta\left(s\right)\right)=\int_{\mu_2(y)}^{x}g_n\left(x,t,\zeta\left(s\right),\eta\left(s\right)\right)dt+$$

$$+q_0\left(x,y,\zeta\left(s\right),\eta\left(s\right)\right),$$

$$q_0(x,y,\zeta(s),\eta(s))=-\frac{\eta'(s)}{\pi}\times$$

$$\times\ln\left(\left(x-y+\mu_2\left(y\right)-\zeta\left(s\right)\right)^2+\left(\mu_2\left(y\right)-\eta\left(s\right)\right)^2\right)-$$

$$-\frac{\eta'(s)}{\pi}\left(arctg\frac{\left(x-\zeta\left(s\right)\right)+\left(y-\eta\left(s\right)\right)}{\left(x-\zeta\left(s\right)\right)-\left(y-\eta\left(s\right)\right)}-\right.$$

$$\left.-arctg\frac{\mu_2\left(y\right)+\left(x-\zeta\left(s\right)\right)-y-\eta\left(s\right)}{\left(x-\zeta\left(s\right)\right)-\left(y-\eta\left(s\right)\right)}\right).$$

u(*x*,*y*) ni topish uchun (1.7) tenglamani quyidagi chegaraviy shart bilan yechamiz

$$u\left(x,y\right)=f_1\left(x,y\right),\ \left(x,y\right)\in\Gamma_2.$$

Bu masalani yechimi quyidagi formula bilan aniqlanadi

$$u(x,y)=\int_{\mu_2(y)}^{x}W\left(t,y\right)dt+f_1\left(x,y\right)\ .\ (1.19)$$

So'ngi ifodadan

$$u_s\left(x,y\right)=f_1'\left(x,y\right),\ \left(x,y\right)\in\Gamma_2\ (1.20)$$

tenglikdan foydalanib $f_3(x,y)$ni topamiz.

$$u_x=W\left(x,y\right)$$

formulaga asosan

$$u_y=\mu_2'\left(y\right)W\left(\mu_2\left(y\right),y\right)+\int_{\mu_2(y)}^{x}W_y\left(t,y\right)dt+f_{1y}\left(x,y\right)\Big|$$

tengliklar o'rinli va olingan hosilalarni (1.20) ga qo'yib , $u_s=x_su_x+y_su_y$ tenglik va (1.18) dan foydalanib

$$f_3\left(\mu_2\left(y\right),y\right)=\frac{K_2\left(y\right)}{\mu\left(y\right)} \quad (1.21)$$

ifodani topish mumkin.

Bunda

$$K_2\left(y\right)=K_1\left(\mu_2\left(y\right),\left(y\right)\right)-\left(x_s+1\right)f_{1s}\left(\mu_2\left(y\right),\left(y\right)\right)+y_s f_{1y}\left(\mu_2\left(y\right),\left(y\right)\right),$$

$$K_1\left(x,y\right)=\frac{1}{\pi}\int_{\Gamma}\eta'\left(s\right)\ln\left(\left(x-\xi\right)^2+\left(y-\eta\right)^2\right)+f_1\left(x,y,\xi\left(s\right),\eta\left(s\right)\right)\omega\left(s\right)ds,$$

$$\mu\left(y\right)=x_n\left(x_s-y_s\mu_2'\left(y\right)\right).$$

(1.21) ni (1.18) ga qo'ysak u holda $W\left(x,y\right)$ funksiya to'liq topiladi. Shunga asosan (1.19) da $u\left(x,y\right)$ funksiya aniqlanadi.

Teorema isbotlandi.

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