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Polar Coordinates in Engineering: Methods for Determining the Length of Curved Geometric Shapes

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Abstract: This article is dedicated to an in-depth study of the practical significance of the polar coordinate system for determining the lengths of curved geometric shapes. Within the context of technical disciplines and engineering education, the article thoroughly describes the mechanisms of converting from polar to Cartesian coordinates, as well as differential and integral formulas used for calculating curve length. Specific examples are provided for calculating the lengths of typical geometric figures such as circles, cardioids, and ellipses, demonstrating the application of this theoretical knowledge in solving real engineering problems. This approach is widely used in the design of mechanical structures, automotive engineering, and aerospace systems. The article emphasizes that this work, connecting mathematical analysis and practical technical knowledge, holds significant value for students not only from an academic but also from a professional development perspective.

Keywords: Polar coordinates, curve length, integral calculus, geometric modeling, engineering design, Cartesian coordinates, mathematical analysis, technical applications, industrial geometry, mechanical design.

It is important for students of the Andijan Institute of Mechanical Engineering and technical disciplines to study methods for calculating the length of curves using the polar coordinate system. [1,2]. We will use integrals and formulas used in calculating the length of curves and their transformation into Cartesian coordinates [5,6]. We will show how to calculate the lengths of geometric shapes such as circles, cardioids, and ellipses with examples. This system is widely used in the design of mechanical structures and other technical systems, while combining mathematical and technical knowledge, which is of practical importance for students [3,4].

Geometry and mathematical analysis play an important role, especially in the field of engineering. For students of the Andijan Institute of Mechanical Engineering, this topic is of great importance not only theoretically, but also practically [8]. In the design of large structures, machines, mechanical systems and their parts, the need to determine the length of geometric shapes, in particular curves, constantly arises. Such problems are used in many engineering fields, including mechanical engineering, automotive engineering, aerospace and other engineering [7,9]. The polar coordinate system serves as a very convenient tool in these fields, because it simplifies complex geometric shapes, such as circles, cardioids, and ellipses, making it easier to calculate their lengths.

The polar coordinate system is used to describe the location of a point on a two-dimensional sphere. In this system, the position of a point is specified by a radius $\rho$and an angle θ. Calculating the length of a curve given in polar coordinates is particularly useful in mechanical engineering or other technical fields, such as designing mechanical systems, determining the shape of materials, and determining their cross-section. In this article, we will look at how to calculate the length of a curve given in polar coordinates and how to apply these calculations in engineering.

Transition to polar coordinate system and Cartesian coordinates. Although the polar coordinate system is very useful for describing simple geometric shapes, it is often necessary to convert them to Cartesian coordinates, since in many cases it is more convenient to use Cartesian coordinates to calculate the

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length of a line or determine its location. The location of a point in polar coordinates is given by $\boldsymbol{\rho}$ and $\boldsymbol{\theta}$:

Let the equation of the curve in polar coordinates be given as:

$$\rho = f(\theta) \quad (1)$$

The formula for converting from polar coordinates to Cartesian coordinates is:

$x = \rho\cos\theta\,, y = \rho\sin\theta$ or if we use (1):

$$x = f(\theta)\cos\theta\,, y = f(\theta)\sin\theta$$

Looking at these equations as parametric equations of the curve,

$$\frac{dx}{d\theta} = f'(\theta)\cos\theta - f(\theta)\sin\theta\,, \frac{dy}{d\theta} = f'(\theta)\sin\theta - f(\theta)\cos\theta$$

In that case

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = (f'(\theta))^2 + (f(\theta))^2 = {\rho'}^2 + \rho^2$$

So,

$$S = \int_{\theta_2}^{\theta_1} \sqrt{{\rho'}^2 + \rho^2}. \quad (2)$$

Examples.

1) $x^2 + y^2 = r^2$Calculate the length of the circle.

Solution. First, we calculate the length of the quarter of the circle that lies in quadrant 1. Then $AB$the equation of the arc is

$$y = \sqrt{r^2 - x^2}, \frac{dy}{dx} = -\frac{x}{\sqrt{r^2 - x^2}}$$

$$\frac{1}{4}S = \int_0^2 \sqrt{1 + \frac{x^2}{r^2 - x^2}}dx = \int_0^2 \frac{r}{\sqrt{r^2 - x^2}}dx = r \cdot \arcsin\frac{x}{2}\Big|_0^2 = r \cdot \frac{\pi}{2}$$

The length of the entire circle is: $S = 2\pi r$.

2) $\rho = a(1 + \cos\theta)$ Find the length of the cardioid. The cardioid is symmetrical about the polar axis. By changing θ the polar angle 0from $\pi$to , we find half the desired length . We use formula (2), where

$$\rho' = -\operatorname{a}\sin\theta$$

$$S = 2 \cdot \sqrt{a^2\big((1 + \cos\theta)\big)^2 + a^2\sin^2\theta\, d\theta} = 2a\sqrt{2 + 2\cos\theta}\, d\theta =$$

$$= 4a \cdot \int_0^\pi \cos\frac{\theta}{2}d\theta = 8a\sin\frac{\theta}{2}\Big|_0^\pi = 8a \cdot 1 = 8a.$$

3) $x = a\cos t\,, y = b\sin t\,, 0 \le 1 \le 2\rho$Calculate the length of the ellipse, where $a > b$.

Solution. We use formula (2). First, we calculate 1/4 of the arc length.

$$\frac{S}{4} = \int_0^{\frac{\pi}{2}} \sqrt{a^2\sin^2 t + b^2\cos^2 t}\, dt = \int_0^{\frac{\pi}{2}} \sqrt{a^2(1 - \cos^2 t) + b^2\cos^2 t}\, dt =$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{a^2 - (a^2 - b^2)\cos^2 t}\, dt = a \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{a^2 - b^2}{a}\cos^2 t}\, dt =$$

$$= a \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \cos^2 t}\, dt.$$

In this

$$k = \frac{\sqrt{a^2 - b^2}}{a} < 1.$$

So,

$$S = 4a \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \cos^2 t}\, dt.$$

The polar coordinate system is a widely used system in geometric and mathematical modeling, especially effective in depicting complex shapes, such as circles, ellipses, cardioids, and other lines. For students of the Andijan Institute of Mechanical Engineering and specialists in technical fields, this system is of great importance not only theoretically, but also for practical purposes. This system allows you to determine lines and shapes in various mechanical, automotive, and machine-building structures, calculate their lengths, and analyze the operation of mechanical systems and components.

For students of the Andijan Institute of Mechanical Engineering, especially in technical fields, the fundamentals of engineering and modern calculation methods are important [10]. The study of the polar coordinate system, this branch of mathematics, and its application in practice combine technical and mathematical knowledge. This, in turn, allows students of technical disciplines to successfully demonstrate themselves not only theoretically, but also practically.

It can be concluded that the polar coordinate system and methods for calculating the length of curves using it are of great importance not only in the educational process, but also in real industrial and technical fields. For students of the Andijan Institute of Mechanical Engineering, this knowledge will help them achieve success in their professional activities by studying practical calculation methods. Therefore, mastering these methods and working with them will serve as a solid foundation for future professional success in engineering and technology.

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